Moduli and Damping Factors for Dynamic Analyses of Cohesionless Soils

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Abstract: Data are presented concerning the shear modulus and damping ratios of sands and gravelly soils as determined by laboratory and field tests. A simple relationship is proposed to relate the shear modulus of a cohesionless soil to a modulus stiffness coefficient, which is a soil property and depends on the characteristics of the soil, and the effective mean principal stress at any point in the soil. Values for the modulus coefficient at low strains are suggested, and it is shown that these values for sands can be estimated from the standard penetration resistance of the sand. Values for gravels are generally greater than those for sands by factors ranging from 1.35–2.5. Suggestions are also made for determining the variation of shear modulus with shear strain and the damping ratios for both sandy and gravelly soils.

Introduction

Much progress has been made in recent years in the development of analytical procedures for evaluating the response of soil deposits and earth structures under seismic loading conditions. Successful application of such procedures for determining ground response in specific cases, however, is essentially dependent on the incorporation of representative soil properties in the analyses. Thus considerable effort has also been directed toward the determination of soil properties for use in these analytical procedures.

In cases of ground response involving no residual soil displacements, the response is determined mainly by the shear modulus and damping characteristics of the soil under reasonably symmetrical cyclic loading conditions. In such cases analyses are often made using the equivalent-linear analysis method in which the moduli and damping factors used in the analysis are compatible with the strains developed in the soil deposit or earth structure. Because most soils have curvilinear stress-strain relationships as shown in Fig. 1, the shear modulus is usually expressed as the secant modulus determined by the extreme points on the hysteresis loop, while the damping factor is proportional to the area inside the hysteresis loop. It is readily apparent that each of these properties will depend on the amplitude of the strain for which the hysteresis loop is determined (see Fig. 1), and thus both shear moduli and damping factors must be determined as functions of the induced strain in a soil specimen or soil deposit.

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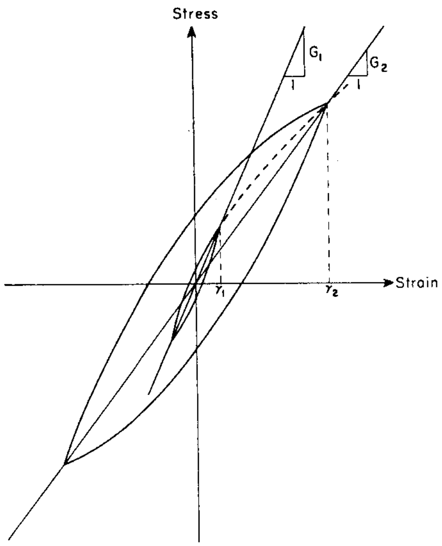

FIG. 1.—Hysteretic Stress-Strain Relationships at Different Strain Amplitudes

The purpose of this paper is to review available information on the dynamic shear moduli and damping factors for sands under loading conditions similar to those shown in Fig. 1, to present new data on similar properties for gravels, and to present the results in a form that will provide a useful guide in the selection of soil characteristics for analysis purposes.

Previous Studies of Moduli for Sands

Hardin and Drnevich (1972), Krizek (1974), and Kuribayashi et al. (1974) have shown clearly that modulus values for sands are strongly influenced by three main factors: (1) The confining pressures; (2) the strain amplitude; and (3) the void ratio (or relative density). Thus for practical purposes, a convenient relationship between the shear modulus and the confining pressure is provided by the simplified equation (Seed and Idriss, 1970)

$$G = 1{,}000 K_2 (\sigma'_m)^{1/2} \quad (1)$$

in psf units, so that the influence of void ratio and strain amplitude can be expressed through their influence on the soil modulus coefficient, K_2. For any sand, this coefficient has a maximum value, $(K_2)_{max}$, at very low strains of the order of $10^{-4}\%$. Values of $(K_2)_{max}$ determined by laboratory tests have been found to vary from about 30 for loose sands to about 75 for dense sands.

Many investigators (Weissman and Hart, 1961; Richart et al., 1962; Drnevich et al., 1966; Silver and Seed, 1969; Hardin and Drnevich, 1972; Seed and Idriss, 1970; Shibata and Soelarno, 1975; Iwasaki et al., 1976;

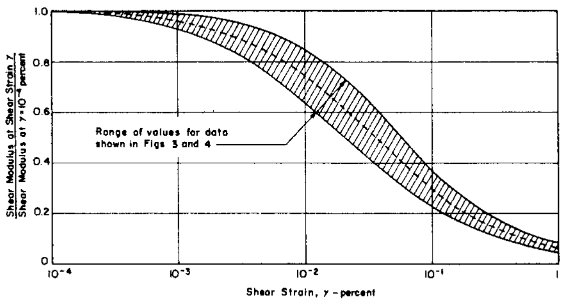

FIG. 2.—Variation of Shear Modulus with Shear Strain for Sands (after Seed and Idriss, 1970)

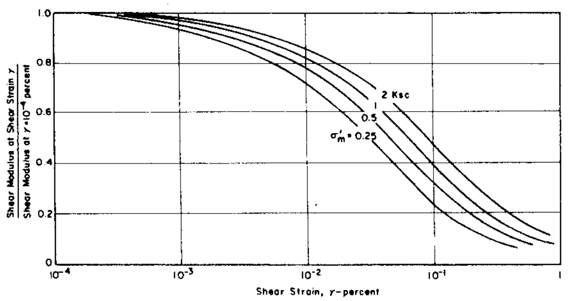

FIG. 3.—Variation of Shear Modulus with Shear Strain for Sands (after Iwasaki et al., 1976)

Kokusho, 1980; Prakash and Puri, 1981; Sherif and Ishibashi, 1977; Edil and Luh, 1978; and others) have studied the relationship between shear modulus (or the shear modulus coefficient, K_2) and shear strain amplitude. Most of these studies have shown that when test data are plotted to show the variation with shear strain of the ratio of shear modulus at strain γ to shear modulus at a shear strain of 10^{-4}% the results fall within the relatively narrow band shown in Fig. 2. Thus a close approximation to the modulus versus shear strain relationship for any sand can be obtained by determining the modulus at a very low strain level, e.g. wave propagation methods in the field, and then reducing this value for other strain levels in accordance with the results indicated by the average (dashed) line in Fig. 2.

The studies by Prakash and Puri (1981) using in-situ tests, indicate that for silty sands the modulus attenuation curve may be slightly flatter than that shown in Fig. 2, but the difference is relatively small.

It should also be noted that the studies by Hardin and Drnevich and the experimental results of Shibata and Soelarno (1975) and Iwasaki et al. (1976) show that the modulus attentuation curve for sands is influenced slightly by the confining pressure. The experimental results of Iwasaki et al. are shown in Fig. 3. Thus where more refined analyses are required, it may be more appropriate to use a family of curves similar to those shown in Fig. 3 to evaluate the response of sand deposits.

Comparison of Values of $(K_2)_{max}$ for Sands Determined by Laboratory and Field Tests

As noted above, values of the modulus coefficient $(K_2)_{max}$ based on laboratory tests generally range from about 30–75. The results of a number of determinations of shear moduli for sands at very low strain levels by means of in-situ shear wave velocity measurements are summarized in Table 1; the six investigations for dense to extremely dense sands (excluding clayey and partly cemented sands) give values for $(K_2)_{max}$ ranging from 44–86. Thus there appears to be good general agreement between the results of laboratory and in-situ investigations.

Further evidence of this result is provided by studies by Ohta and Goto (1976). On the basis of numerous shear wave velocities measured in the field, these investigators presented the following equation:

$$v_s\left(\frac{m}{s}\right) = 69N_j^{0.17}\,D^{0.2}(m) \times F_1 \times F_2 \quad (2)$$

where N_j = SPT N-value as measured in Japanese practice; D = depth of soil below ground surface; F_1 = a factor, depending on the nature of

TABLE 1.—Shear Moduli[a] of Sands Based on In-Situ Shear Wave Velocity Measurements

Soil (1)	Location (2)	Depth (ft) (3)	K_2 (4)
Loose moist sand	Minnesota	10	34
Dense dry sand	Washington	10	44
Dense saturated sand	So. California	50	58
Dense saturated sand	Georgia	200	60
Dense saturated silty sand	Georgia	60	65
Dense saturated sand	So. California	300	72
Extremely dense silty sand	So. California	125	86
Dense dry sand (slightly cemented)	Washington	65	166
Moist clayey sand	Georgia	30	119

[a]Shear modulus, $G = 1{,}000K_2(\sigma'_m)^{1/2}$ psf.

TABLE 2.—F_2 Factors for Various Soil Types

Soil type (1)	Factor F_2 (2)
Clay	1.0
Fine sand	1.09
Medium sand	1.07
Coarse sand	1.14
Sandy gravel	1.15
Gravel	1.45

the soil, having a value of 1 for alluvial deposits and 1.3 for diluvial deposits; and F_2 = a factor, depending on the nature of the soil as shown in Table 2.

Thus for sands and sandy gravel deposits, the average value of F_1 is about 1.15 and the average value of F_2 is close to 1.1 so that the product of F_1F_2 is typically very close to 1.25. Converting the results to fps units and U.S. practice in the measurement of N values, Eq. 2 reduces to

$$v_s = 220N_{60}^{0.17}D^{0.2} \text{ fps} \quad (3)$$

where N_{60} = N-value measured in SPT test delivering 60% of the theoretical free-fall energy to the drill rods; and D = depth of soil in feet.

Actually, due to the small power of N_j in the original equation, the difference in SPT N-values can be neglected for all practical purposes.

$$\text{Since} \quad G_{max} = \frac{\gamma}{g}v_s^2 \quad (4)$$

Eq. 3 provides a correlation between G_{max} and SPT N-value, based on field test data as follows. Assuming γ = 120 pcf, substitution of Eq. 3 in Eq. 4 leads to

$$G_{max} = 180 \times 10^3 \cdot N_{60}^{0.34} \cdot D^{0.4} \text{ psf} \quad (5)$$

If the water table is at a relatively shallow depth below the ground surface, the effective stress at depths below 10 ft may be expressed approximately by

$$\sigma_0' \simeq 62.5D \text{ psf} \quad (6)$$

$$\text{Thus} \quad D \simeq \frac{\sigma_0'}{62.5 \text{ ft}} \quad (7)$$

Substituting Eq. 7 into Eq. 5 leads to

$$G_{max} \simeq 35 \times 1{,}000N_{60}^{0.34}(\sigma_0')^{0.4} \text{ psf} \quad (8)$$

Noting that $N = N_1/C_N$ leads to

$$G_{max} \simeq 35 \times 1{,}000(N_1)_{60}^{0.34}\frac{(\sigma_0')^{0.4}}{C_N^{0.34}} \text{ psf} \quad (9)$$

It can readily be shown that with a high degree of accuracy for effective stresses up to 6,000 psf

TABLE 3.—Values of $(K_2)_{max}$ for Various N_1 Values

$(N_1)_{60}$ (1)	$(K_2)_{max}$ (2)
5	34
8	40
10	43
18	52
28	61
44	71

$$\frac{(\sigma_0')^{0.4}}{C_N^{0.34}} \simeq 0.47(\sigma_0')^{0.5} \quad (10)$$

and thus, Eq. 9 may be rewritten as

$$G_{max} \simeq 16.5 \times 1{,}000(N_1)_{60}^{1/3}(\sigma_0')^{1/2} \text{ psf} \quad (11)$$

For normally consolidated deposits $\sigma_m' \simeq 0.65\sigma_0'$ and thus

$$G_{max} \simeq 1{,}000 \times 20(N_1)_{60}^{1/3}(\sigma_m')^{1/2} \text{ psf} \quad (12)$$

It may be noted that this equation has the same form as Eq. 1 provided

$$(K_2)_{max} \simeq 20(N_1)_{60}^{1/3} \quad (13)$$

Values of $(K_2)_{max}$ determined from Eq. 13 for N_1 values ranging from N_1 = 5 (loose sands) to N_1 = 44 (very dense sands) are shown in Table 3. These values for $(K_2)_{max}$ are in the same range as those discussed previously, providing further confirmation of the similarity in values of $(K_2)_{max}$ for laboratory and field determinations.

Damping Ratios for Sands

Studies performed by Hardin and Drnevich (1972), Seed and Idriss (1970), and others have shown that although such factors as grain size characteristics, degree of saturation, void ratio, lateral earth pressure coefficient, angle of internal friction, and number of stress cycles have minor effects on the damping ratios for sands, the main factors affecting the damping ratio are the strain level induced in the sand and the effective confining pressure to which it is subjected. The influence of confining pressure, as determined by two studies, is shown in Figs. 4 and 5. For pressures less than about 500 psf, the effect of pressure changes may be significant, but excluding these very low pressures, which represent conditions in the top few feet of soils, the effect of variations in pressure is very small compared with the effect of shear strain. An average damping ratio versus shear strain relationship determined for an effective vertical stress of 2,000–3,000 psf would appear to be adequate for many practical purposes. Considering the potential scatter of test data for damping ratios (see Fig. 6), even those obtained by the same investigator using the same test procedure, the adoption of such an average relationship may be even more justified.

Approximate upper and lower bound relationships between damping ratio and shear strain are shown by the dashed lines in Fig. 6 and a representative average relationship for all of the test data is shown by the solid line. This average relationship is likely to provide values of damping ratio with sufficient accuracy for many practical purposes.

The curves in Fig. 6 also provide a basis for evaluating the relationship between damping ratio and strain for particular sands for which limited test data is available. If the value of damping ratio at a strain level of 0.1–0.5% is determined, the probable damping ratios at other strains can be closely approximated by drawing a line through the known data point parallel to the curves shown in Fig. 6.

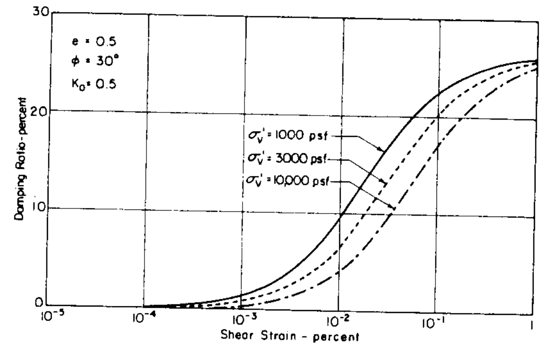

FIG. 4.—Influence of Confining Pressure on Damping Ratio of Saturated Sand (Based on Hardin and Drnevich Expressions)

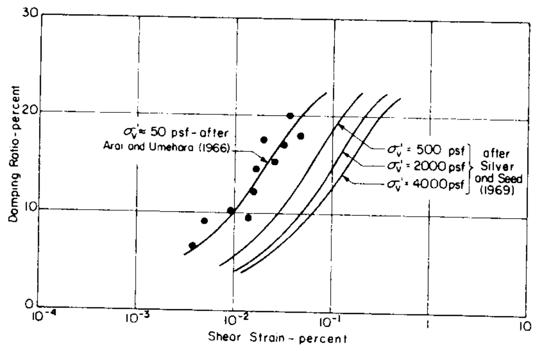

FIG. 5.—Influence of Confining Pressure on Damping Ratio of Dry Sand

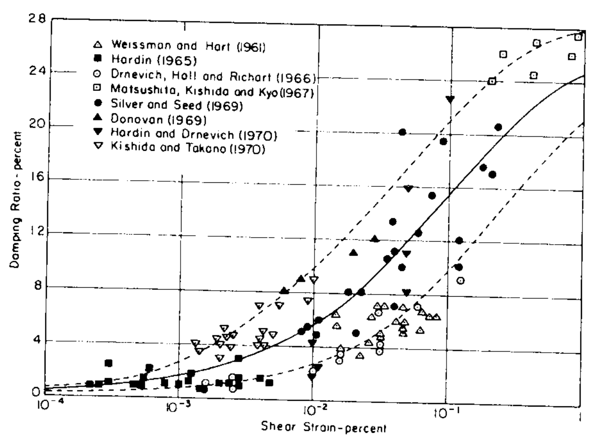

FIG. 6.—Damping Ratios for Sands

Probably because of the large diameter of test specimens required (about 12 in.), there have been virtually no laboratory studies of the shear moduli and damping ratios for gravelly soils. It has long been recognized, however, that shear wave velocities are significantly higher in gravels than in sands, indicating that the modulus coefficient K_2 will also be higher. The results of several modulus determinations for gravelly soils, based on in-situ shear wave velocity measurements, are summarized in Table 4, from which it may be seen that at small strain levels, modulus values are between 1.25 and 2.5 times greater for gravelly soils than for dense sands.

To supplement the meager data available concerning the shear modulus-strain relationship for gravelly soils, a comprehensive series of tests was performed on 12-in. diameter samples of several different types of gravel in the University of California Rockfill Testing Facility. Tests were performed on isotropically-consolidated samples of gravelly soils under undrained cyclic loading conditions.

Materials Tested.—The soils tested included 1-1/2–3/4 in. uniformly graded Oroville gravel, well-graded Oroville gravel, well-graded Pyramid gravel, well-graded gravel prepared from Venado sandstone, and the Livermore natural gravel deposit. The gradations of the field materials and the modeled gradations used in the test program are shown in Fig. 7.

The Oroville gravel was prepared from the shell material used for the Oroville Dam. The material was well-rounded, while the fine particles were subrounded to subangular. The particles (which were mostly amphibolite) were hard and it was very difficult, almost impossible, to break the medium gravel-size particles with a hammer. The specific gravity for the portion of the materials larger than 1/4 in. in size was about 2.94, and that for the portion of the materials finer than 1/4 in. was about 2.86, indicating a change of mineralogy with size.

The Pyramid Dam material is the rockfill material for the shell section of the Pyramid Dam in Southern California. The rockfill material was produced by quarry blasting, and the individual particles, composed of argillite, were very angular in shape. The individual rock particles could

TABLE 4.—Shear Moduli[a] of Gravelly Soils Based on In-Situ Shear Wave Velocity Measurements

Soil (1)	Location (2)	Depth (ft) (3)	K_2 (4)
Sand, gravel, and cobbles with little clay	Caracas	200	90
Dense sand and gravel	Washington	150	122
Sand, gravel and cobbles with little clay	Caracas	255	123
Dense sand and sandy gravel	Southern California	175	188

[a]Shear modulus $G = 1{,}000K_2(\sigma_m')^{1/2}$ psf.

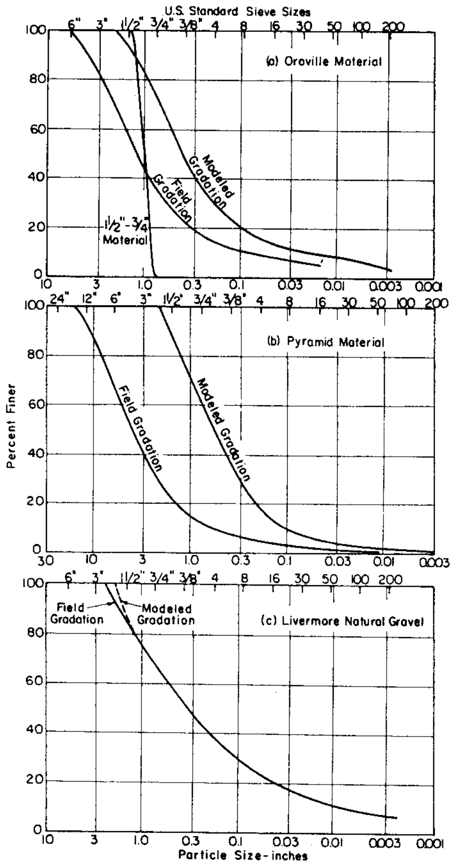

FIG. 7.—Grain Size Distribution Curves for Field and Modeled Gradations

be broken into several pieces with a hammer.

The Venado sandstone was obtained from a medium to thick-bedded sandstone in the Upper Cretaceous Venado formation exposed about 1,500 ft down-stream from Sites damsite in California. After blasting, large pieces of unweathered sandstone, up to 3 ft minimum dimension, were selected for crushing. These large pieces of sandstone were fine-medium grained, well-cemented, and light gray in color. The individual particles after crushing were very angular and comparatively weak so that they could be broken into several pieces and powder with a hammer. The material was much softer than the Oroville Dam material and a little softer than the Pyramid Dam material.

The Livermore natural gravel deposit was obtained from the flood plain of Livermore Valley, one mile east of Pleasanton, 35 miles east of Oakland, California. There was little variation in gradation in the deposit, which averaged about 56% gravel, 36% sand, and 8% clay and silt. About 98% of the gravel was minus 3 in. There was some deficiency of 30 and

TABLE 5.—Specific Gravities and Maximum and Minimum Void Ratios of Soils Tested

Name (1)	Size (2)	Specific gravity (3)	e_{max} (4)	e_{min} (5)
Oroville Dam material	1-1/2 in. to 3/4 in.	2.94	0.81	0.52
Oroville Dam material	2 in. to -No. 200	2.90	0.46	0.176
Pyramid Dam material	2 in. to -No. 200	2.62	0.737	0.366
Venado sandstone	2 in. to -No. 200	2.74	0.923	0.435
Livermore natural deposit	2 in. to -No. 200	2.65	0.455	0.166

50-mesh material. The individual particles were well-rounded to rounded, relatively hard and they were very difficult or almost impossible to break with a hammer.

A more detailed description of the materials has been presented by Marachi, Chan, Seed, and Duncan (1969). The specific gravities and maximum and minimum void ratios of the various soils tested are given in Table 5. The method of determination of relative densities is described by Wong, Seed, and Chan (1974).

Method of Testing.—In this investigation the shear moduli and damping characteristics of the soils were determined from the hysteretic stress-strain relationships determined by cyclic undrained triaxial tests. For each loading cycle, a hysteresis loop was plotted. The equivalent modulus was obtained from the secant modulus, which represented the average modulus of the loop. The equivalent damping ratio, λ_{eq}, at shear strain γ was determined from the area inside the hysteresis loop using standard procedures (Seed and Idriss, 1970). Since the hysteresis loops are a function of the maximum strain applied, both the equivalent modulus and the equivalent damping ratio are strain-dependent. From the values of shear modulus determined in this way, values of the modulus coefficient K_2 were evaluated.

Since K_2 is independent of the confining pressure, tests were conducted to determine the influence of strain amplitude and relative density on the K_2 parameter for several types of gravelly materials. Specimens of different densities for each material were prepared and consolidated under an initial effective confining stress of 2 kg/cm^2. Each specimen was then subjected to a very small axial strain amplitude (on the order of ±0.0003%) for six cycles without drainage. The pore water pressure, which built up slightly during the application of the strain cycles, was released after the sixth cycle. Some time was allowed for the sample to reach an equilibrium state before another six strain cycles of about twice the original amplitude was applied. This was continued until an axial strain amplitude of ±0.2% was reached.

The volume of a specimen decreased when the pore pressure was released by opening the drainage valve and thus the sample became a little denser. The change in volume depended on the axial strain amplitude and the number of strain cycles applied. It was found that the amount of change in density was negligible for small strain amplitudes and was still very small even for strain amplitudes up to ±0.2% if the number of strain cycles was limited. However, the value of K_2 was much less affected by density for higher strain amplitudes. It is therefore believed that the re-use of samples for higher strain amplitudes still gives reasonably good results if the number of strain cycles applied is limited. In order to investigate this more fully, however, a few samples were subjected directly to high strain amplitudes after consolidation. Both their moduli and damping values were found to be very close to those obtained from samples which were previously subjected to smaller strain cycles.

It was also observed that both G and σ'_m varied to some extent with increasing number of cycles for each strain amplitude. For comparison purposes, it was considered that the most appropriate values of K_2 were computed for the initial effective confining stress σ'_m, and the shear modulus G_e at the fifth cycle, since this was considered to be a representative average for many earthquakes.

Test Results for Well-Graded Gravels.—Test data for samples of well-graded Oroville Dam material, prepared at different relative densities, are shown in Fig. 8. It may be noted that: (1) The value of K_2 decreases

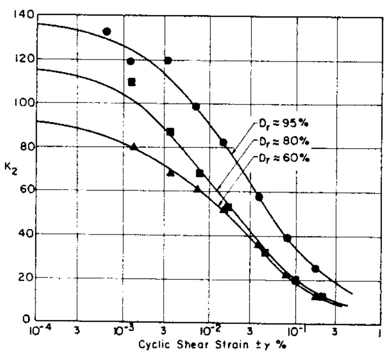

FIG. 8.—Shear Moduli of Well-Graded Oroville Material

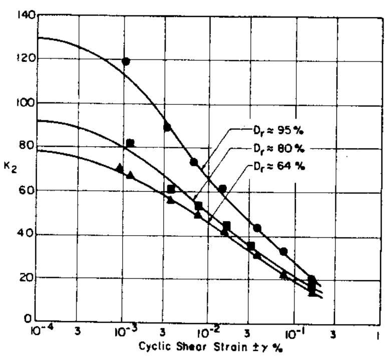

FIG. 9.—Shear Moduli of Model-Graded Pyramid Material

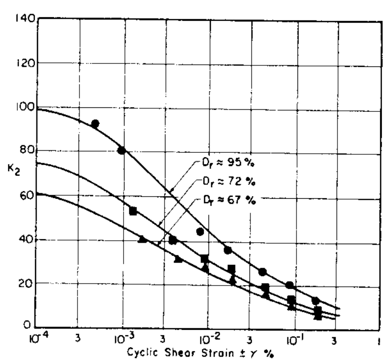

FIG. 10.—Shear Moduli of Well-Graded Venado Sandstone

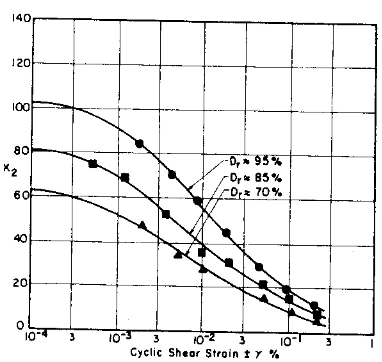

FIG. 11.—Shear Moduli of Well-Graded Livermore Natural Gravel

markedly as the cyclic shear strain increases; and (2) the value of K_2 increases with an increase in the relative density.

Similar tests were performed for the gravels prepared from the Pyramid Dam material and from the Venado sandstone. The measured values of K_2 are shown in Figs. 9 and 10, respectively. It should be noted that the Pyramid Dam material and the Venado sandstone were tested using the same gradation as shown in Fig. 7(*b*). As discussed above, the Pyramid Dam material is somewhat stronger than the Venado sandstone and this is reflected by the higher values of K_2 for this material. Finally the values of K_2 for the well-graded Livermore gravel are shown in Fig. 11.

The values of K_2 for the four types of material, together with typical values for sands (see Fig. 2), are summarized in Fig. 12 for gravels hav-

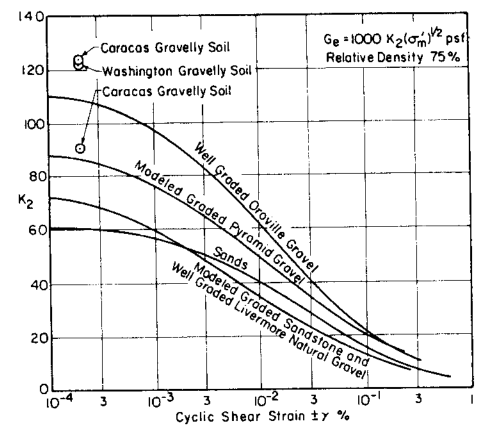

FIG. 12.—Comparison of Shear Moduli for Gravelly Soils and Sands at $D_r \simeq 75\%$

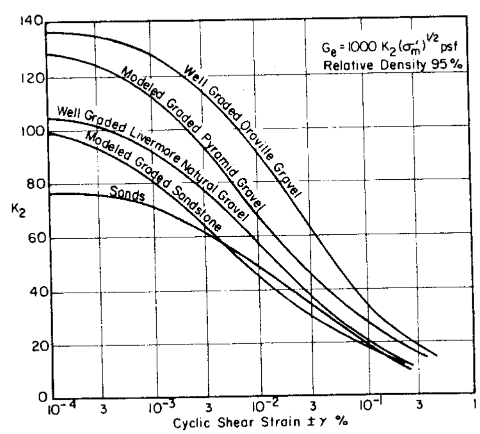

FIG. 13.—Comparison of Shear Moduli for Gravelly Soils and Sands at $D_r \simeq 95\%$

ing a relative density of about 75% and in Fig. 13 for gravels at about 95% relative density. It was found that the Oroville Dam gravel was the stiffest of those tested. Values of K_2 for this gravel were about twice the average value for sands. However, values of K_2 for both the Venado sandstone gravel and Livermore natural deposit were found to be very close to those for sands. The K_2 values for the Pyramid Dam material were located between those for the Oroville Dam material and those for sands.

It is interesting to note that the values of $(K_2)_{max}$ for these four different gravels range from about 75–135 depending on the relative density of the soil and the hardness of the particles. The lower bound of these values is significantly lower than that for the field tests shown in Table 5 and could have been attributed to the soft particles of Venado sandstone were it not for the generally similar values determined for the Livermore natural gravel. On the whole it would appear that values of $(K_2)_{max}$ for relatively dense well-graded gravels are likely to range from about 80–180, compared with a range of about 55–80 for sands. However, modulus attenuation with strain for such gravels is quite similar to that for sands, as illustrated by the normalized modulus versus strain

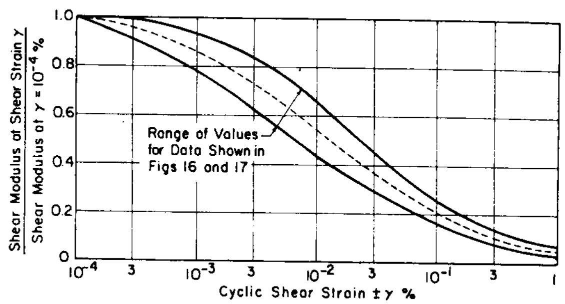

FIG. 14.—Variation of Shear Modulus with Shear Strain for Gravelly Soils

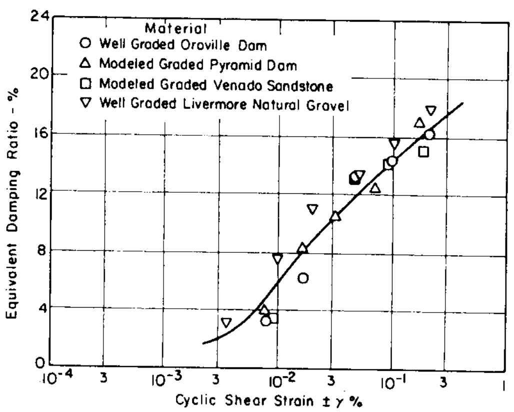

FIG. 15.—Equivalent Damping Ratios for Gravelly Soils at $D_r \simeq 80\%$

plot for the gravel test data shown in Fig. 14. It may be seen that the modulus attenuation curves in the figure are slightly flatter than those for sands shown in Fig. 2.

It is interesting to note that the results of tests on uniformly graded Oroville gravel with particles in the size range 3/4–1-1/2 in. showed modulus values only about 10% lower than those for well-graded Oroville gravel at low shear strain levels (less than about $10^{-2}\%$) and even smaller differences at strain levels above $10^{-2}\%$. Gradation does not appear to be a significant factor determining the shear modulus of gravels if this result can be considered to be generally indicative of other gravels.

Equivalent Damping Ratios for Well-Graded Gravelly Soils.—The hysteresis loops at the fifth cycle for each strain amplitude, from which the moduli of the materials were determined in previous sections, were also used to evaluate the equivalent damping ratios of the different materials. Measured values for all materials at a relative density of 80% are

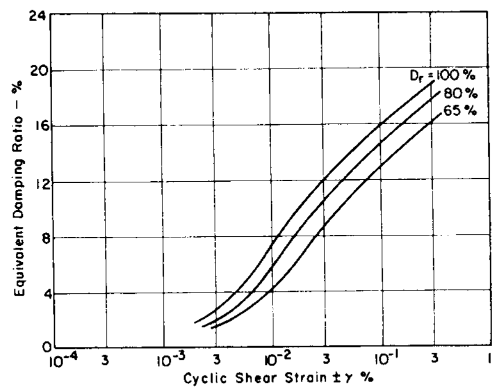

FIG. 16.—Effect of Relative Density on the Damping Ratio versus Strain Relationship for Gravelly Soils

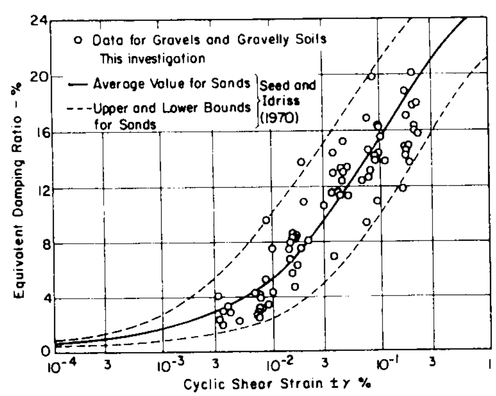

FIG. 17.—Comparison of Damping Ratios for Gravelly Soils and Sands

shown in Fig. 15, from which it would appear that variations with type of material are small.

A careful study of the test data showed that for the well-graded Oroville material, the Pyramid material, and the Livermore sand and gravel, the damping ratios were somewhat higher at the higher densities. Since the damping ratio indicates the amount of energy dissipated during each cycle, it might be expected that dense material would dissipate more energy and, thus, have a higher damping ratio than loose materials. However, this phenomenon was not clearly evident for the Venado sandstone whose damping ratios fell within a small band for all densities.

In order to better explore the effect of density on the damping ratios of the four types of material, Fig. 16 was plotted to show average damping ratios for the four different materials at relative densities of approximately 100%, 80%, and 65%. The influence of relative density on damping ratio is clearly apparent in these data.

The equivalent damping ratios for the uniformly graded Oroville 1-1/2–3/4 in. material were found to be very similar to those determined for the well-graded Oroville material, suggesting that there is no significant effect of gradation on the equivalent damping ratios.

As for the moduli determination, it was found that the equivalent damping ratio was not significantly affected by the number of cycles at very small strain amplitudes. However it decreased to approximately 3/4 its original value after 60 cycles at an axial strain amplitude of ±0.2% were imposed. It should be noted that the equivalent damping ratios at the fifth cycle were used in the previous comparison, since this was considered to be a representative average for most earthquakes.

Comparison of Equivalent Damping Ratios of Gravels and Gravelly Soils with Values for Sands

The relationship between shear strain and equivalent damping ratio for sands, using data from a variety of sources, is shown in Fig. 6. Measured values of damping ratio determined in all tests on the four gravels used in this investigation are shown in Fig. 17, where they are also compared with the range of values for sands shown in Fig. 6. It would appear from these results that damping ratios for gravels are very similar to those for sands.

Conclusions

Based on the studies described in the preceding pages it may be concluded that:

1. For most practical purposes, the dynamic shear moduli of granular soils (sands and gravels) can conveniently be expressed by the relationship (Eq. 1)

$$\boxed{G = 1{,}000 \cdot K_2 \cdot (\sigma'_m)^{1/2}}$$

in psf units where σ'_m = the effective mean principle stress (in psf); and K_2 = a shear modulus coefficient, which is mainly a function of the grain size of the soil particles, the relative density of the soil, and the shear strain developed in the soil. Other characteristics have minor effects on the results and are not usually significant for most practical purposes.

2. A useful guide to the determination of appropriate values of K_2 for use in the above equation is the relationship expressed by Eq. 13

$$(K_2)_{max} \simeq 20(N_1)_{60}^{1/3}$$

3. Values of the modulus coefficient K_2 for gravels are generally greater than those for sands by factors ranging from about 1.35–2.5.

4. The form of the variation of effective modulus with strain shown in Fig. 2 is generally representative of most sands, but the curve for gravels may well be a little flatter than that for sands.

5. Since the modulus attenuation curve for most cohesionless soils is about the same, the shear moduli for any given soil are generally characterized by the modulus coefficient measured at low strains, $(K_2)_{max}$ for that soil. Values of $(K_2)_{max}$ for sands are generally in the range of 30 for very loose sands to about 75 for very dense sands. Values of $(K_2)_{max}$ for relatively dense gravels are generally in the range of about 80–180.

6. Damping ratios for sands and gravels are very similar, and representative values are given by the curves shown in Figs. 6 and 17.

7. Values of damping ratio for sands and gravels are only slightly affected by density and not significantly dependent on the grain size of the particles.

While the moduli and damping ratios for cohesionless soils are undoubtedly influenced to some extent by other factors than those discussed earlier (such as confining pressure, number of stress cycles, degree of saturation, time effects, etc.), it is suggested that the use of the modulus coefficient $(K_2)_{max}$ as discussed in this paper, the modulus attenuation curve shown in Fig. 2, and the damping ratios shown in Figs. 6 and 17 can provide a convenient basis for determining dynamic properties for cohesionless soils which will be sufficiently accurate for many practical purposes.

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QUIESCENT CONSOLIDATION OF PHOSPHATIC WASTE CLAYS

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ABSTRACT: A number of currently employed mathematical formulations used in modeling one-dimensional quiescent consolidation of phosphatic waste clay ponds are reviewed. It is shown theoretically, as well as through an example, that the excess pore water pressure, void ratio, and ground settlements of the models investigated are identical, with the only difference being in the selection of coordinate representation or dependent variables. Correlation between centrifuge prediction of a prototype pond and theory is presented with material parameters obtained from the laboratory. It is concluded that even though the effective stress versus void ratio representation is acceptable, the present laboratory techniques of finding void ratio versus permeability are deficient.

INTRODUCTION

Phosphate is the primary source of phosphorus in inorganic fertilizers with approximately 80% of the United States' requirements and 30% of the world's needs mined in the state of Florida. The matrix of the excavated material is typically composed of 1/3 phosphate, 1/3 granular materials (sand), and 1/3 clays (montmorillonite, attapulgite, illite, and kaolinite) (4). The beneficiation process converts the matrix to a dilute solution from which the phosphate is skimmed, the granular material screened, leaving a dilute clay slurry for disposal. For economical, as well as mining (water recovery) reasons the slurry, which ranges anywhere from 2–6% solids content (solids content = Ws/W), is pumped into large retention ponds and allowed to settle/consolidate. However, since the volume of waste slurry generated from the ore extraction process exceeds the volume originally occupied by the matrix, large above ground earth dikes (anywhere from 3–15 m high) are needed to impound the clays, as shown in Fig. 1.

The adverse impact of this waste disposal technique is: (1) It ties up tremendous quantities of water; and (2) it prevents the development of valuable land (close to 100,000 acres) for agricultural, residential, and/or commercial purposes for many years. As a result, significant effort has been expended in finding the most accurate way to predict the rate of consolidation and the final density (final height) of the waste deposits. Such predictions are necessary to estimate the ultimate storage capacity of a disposal area and the time required to achieve its reclamation.

The physical problem consists of two phases: (1) Settling/sedimentation of the suspended fines, and (2) self-weight consolidation of the sed-

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