Small- and large-strain soil properties from seismic flat dilatometer tests

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ABSTRACT: The seismic flat dilatometer test (SDMT) provides downhole shear wave velocities (V_s) in supplement to conventional inflation readings (p_0 and p_1). Soil stratigraphy and strength parameters are evaluated from the pressure readings while the small-strain stiffness (G_0) is obtained from in-situ V_s profiles. A modified hyperbola is used for modulus degradation in order to span from nondestructive- to intermediate-to failure-level strains. Comparisons are made between the derived stress-strain-strength curves from SDMTs and laboratory tests on soft varved lacustrine clay and residual silty sand. The approximate nonlinear approach is also applied to a load test case study involving shallow foundation displacements.

Keywords: clay, dilatometer, downhole tests, in-situ testing, shear modulus, silty sand, shear strength, strains

1 INTRODUCTION

The seismic flat dilatometer test (SDMT) provides a simple and cost-effective means for determining the initial elastic stiffness at very small-strains and in-situ shear strength parameters at high-strains in natural soil deposits. Pressure readings (p₀ and p₁) from conventional dilatometer tests are taken at 0.2-m depth intervals and used to evaluate failurestate parameters, namely the undrained shear strength (s_u) in clays and effective friction angle (ϕ') in sands. In addition, at 1-m intervals, the small-strain stiffnesss response is obtained by downhole measurements of shear wave velocity (V_s). Thus, three independent readings are obtained from a single sounding. Stiffnesses at intermediate strains can be assessed using a modified hyperbola to degrade and soften the modulus.

A prototype of the SDMT with a triaxial geophone was introduced by Hepton (1988). A recent version uses a single horizontal velocity transducer positioned just above the blade (Kates, 1996). A co-axial cable connects the geophone to an oscilloscope. Figure 1 shows the seismic flat dilatometer equipment and details on the setup and procedure of the test are given by Martin & Mayne (1997, 1998).

Source waves are generated by striking a horizontal plank at the surface that is oriented parallel to the geophone axis. The measured arrival times at successive depths provide pseudo-interval V_s profiles for horizontally-polarized vertically-propagating shear waves.

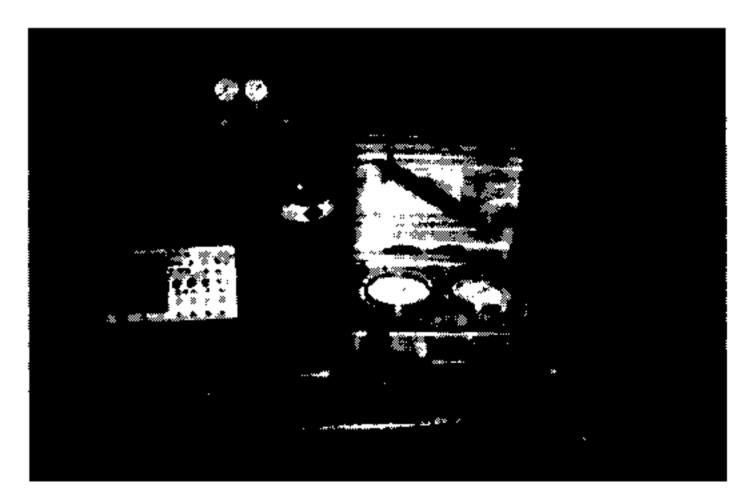


Figure 1. Seismic flat plate dilatometer setup with blade, gauge, velocity geophone, and oscilloscope.

2 SMALL STRAIN STIFFNESS

The small-strain shear modulus (G_0) is determined from the relation:

$$G_0 = \rho(V_s)^2 \tag{1}$$

where $\rho = \gamma_T/g_a = \text{mass density}$, $\gamma_T = \text{total unit weight}$, and $g_a = \text{gravitational acceleration} = 9.8$ evaluated from V_s and depth z (Figure 2):

$$\rho \approx 1 + \frac{1}{0.614 + 58.7(\log z + 1.095)/V_S}$$
 (2)



where z is in metres and V_s in m/s. The database is reported in Burns & Mayne (1996).

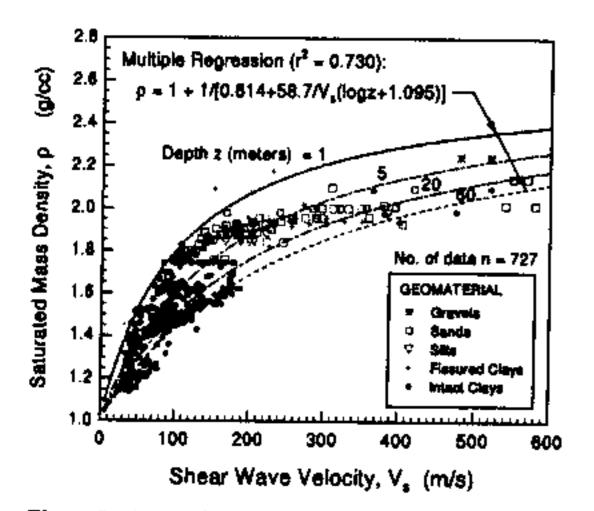


Figure 2. Approximate relation for mass density, shear wave, and depth for geomaterials.

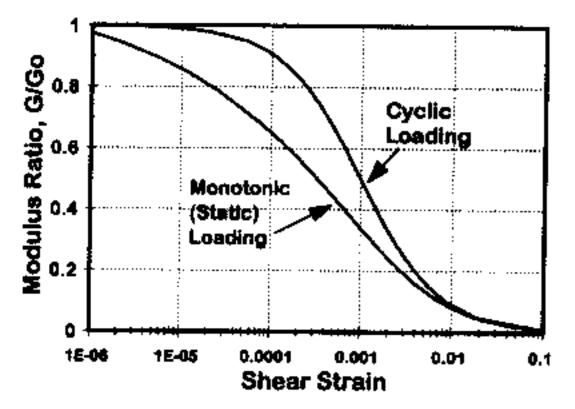


Figure 3. Monotonic and cyclic degradation response with logarithm of shear strain.

The G_0 stiffness at low strains ($\gamma_s < 10^{-6}$) is relevant to both static and dynamic deformation problems. For static monotonic loading of foundations and walls, the characteristic strains of most soil elements are generally small and in the range from 10^{-1} to 10^{-2} percent (Burland, 1989). Thus, the modulus values used for settlement calculations and deformation analyses should correspond to the small- to intermediate-strain region, requiring a modulus degradation scheme such as G/G_0 versus logarithm of shear strain. Empirical schemes are well-known for dynamic loading (e.g., Vucetic & Dobry, 1991), however, the static monotonic case or "backbone curve" (see Figure 3) has been

experimentally shown to have a much faster degradation rate (e.g., LoPresti, et al. 1993). Laboratory torsional shear devices or special instrumented triaxial apparati are available for direct determination of the monotonic behavior, however, the innate and inherent difficulties associated with soil sampling, disturbance effects, reconsolidation to K₀ states, reconstitution, and aging prohibit these tests for routine use in geotechnical practice.

Today, it is convenient to directly obtain the insitu G₀ from crosshole, downhole, surface waves, or suspension logging techniques. Thus, a proper numerical scheme for modulus degradation has become of paramount focus (Tatsuoka & Shibuya, 1992; LoPresti, et al., 1998; Puzrin & Burland, 1998). With G₀ operable at nondestructive strains and shear strength (\tau_{max}) at failure strains (s_u or \(\phi'\)), then a complete stress-strain-strength curve is possible, provided a numerical scheme is provided to connect and bridge the two opposite ends of the strain spectrum.

3 MODULUS DEGRADATION SCHEME

For monotonic static loading, a modified hyperbola (Fahey & Carter, 1993; Fahey, et al. 1994) relates the normalized shear modulus (G/G₀) to mobilized shear stress (τ/τ_{max}) in the form:

$$G/G_0 = 1 - f(\tau/\tau_{max})^g$$
 (2)

where f and g are empirical material parameters controlling the rate of modulus decay. Figure 4 shows the effect of the parameter g on the relation (holding f = 1 constant). A simple hyperbola is given by f = g = 1.

For practical problems, the mobilized shear stress can be considered as the reciprocal of FS, where FS = factor of safety. The corresponding shear strain (decimal) at any value of τ is obtained from:

$$\gamma_i = \tau/G \tag{3}$$

Prior fitting of the modified hyperbola to laboratory data on quartz sands and natural clays has found that f=1 and $g\approx 0.3$ to 0.4 provide reasonable first-guess values for the degradation parameters in non-structured and non-cemented soils (Burns & Mayne, 1996). Similar values have been obtained via backcalculated moduli from full-scale foundation load tests (Mayne & Durnas, 1997). Consequently, for unaged, uncemented, and insensitive geomaterials, a first-order expression for modulus degradation reduces to:

$$G/G_0 = 1 - (\tau/\tau_{max})^{0.3}$$
 (4)

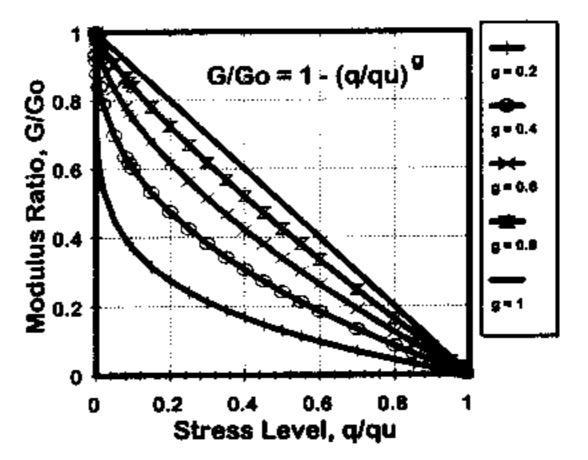


Figure 4. Parametric modulus degradation curves using modified hyperbola form of Fahey et al. (1994).

Experimental studies by Teachavorasinskun et al. (1991) and LoPresti et al. (1993) have shown that the parameter "g" increases slightly with overconsolidation in quartz sands. The latter study also showed similar degradation trends for calcareous sand.

4 STRENGTH

For drained loading, the shear strength (τ_{max}) is determined from the effective friction angle (ϕ') (with effective cohesion intercept c' = 0):

$$\tau_{\text{max}} \approx (\sigma_{\text{vo}}') \tan \phi'$$
 (5)

A more rigorous expression is discussed by LoPresti et al. (1993), however, an evaluation of K₀ is required as well. For clean uncemented quartz sands, Marchetti (1997) posed the following expression as a lower bound to the wedge plasticity solution:

$$\phi' \approx 28^{\circ} + 14.6 \log K_D - 2.1 \log^2 K_D$$
 (6)

For undrained loading of clays, the value of τ_{max} is defined by the undrained shear strength (s_u) and best obtained from the stress history. First, the effective preconsolidation stress (σ_p) is evaluated from the DMT contact pressure, p_0 (Mayne & Bachus, 1989):

$$\sigma_{\rm p}' \approx 0.509(p_0 - u_0)$$
 (7)

where u_0 = hydrostatic pore water pressure. The effective overburden stress (σ_{vo}) is calculated based on cumulative unit weights from mass densities (Eq. 2) and u_0 . Overconsolidation ratios (OCR = σ_p '/ σ_{vo} ') for each test depth are computed

and used in normalized relations (Ladd, 1991) to obtain the undrained shear strength (s_u):

$$s_{u}/\sigma_{vo}' = S(OCR)^{m}$$
 (8)

where S = normally consolidated undrained shear strength (corresponding to simple shear mode) and m = exponent term representing the rate of increase with overconsolidation. Table 1 provides the values of "S" and "m" as suggested by Ladd (1991) for various clay origins.

Table 1. Recommended NC Strength Ratios and OC Exponent Terms for Various Clay Types (Ladd, 1991).

Soil Type	S	m
Sensitive Marine Clays	0.20	1.00
Homogeneous CL and CH Sedimentary Clays of Low to Moderate Sensitivity	0.22	0.80
Northeastern U.S. Varved Clays	0.16	0.75
Sedimentary Silts, Organic Soils, & Clays with Shells	0.25	0.80

5 AMHERST SITE

The modulus degradation methodology has been applied using seismic dilatometer tests obtained at the National Geotechnical Experimentation Site (NGES) at Amherst, Massachusetts. A small portable hydraulic rig provided by the Univ. of Massachusetts was used to push the seismic dilatometer blade to the desired test depths. Complete details concerning the development of the device, field testing, and analysis are given by Kates (1996).

The site is underlain by a thick 30-m deposit of soft lacustrine varved clay where the upper 3 to 4 metres consists of a thin clay fill and desiccated crust (Lutenegger, 1995). Typical index properties of the natural soft clay are: liquid limit ≈ 58 , plasticity index ≈ 32 , natural water content ≈ 65 , clay fraction (CF $< 2\mu$) ≈ 50 %, and OCR ≈ 2 . The groundwater table lies approximately one metre below existing grade.

Two regular DMTs and three SDMTs were performed at the site using a portable rig to push the blade to the desired test depths. A summary of the DMT contact pressures (p₀), expansion pressures (p₁), and shear wave arrival times (t₄) is shown in Figure 5 (Martin & Mayne, 1997). The time difference between each successive event was used to calculate a shear wave velocity (on approximate

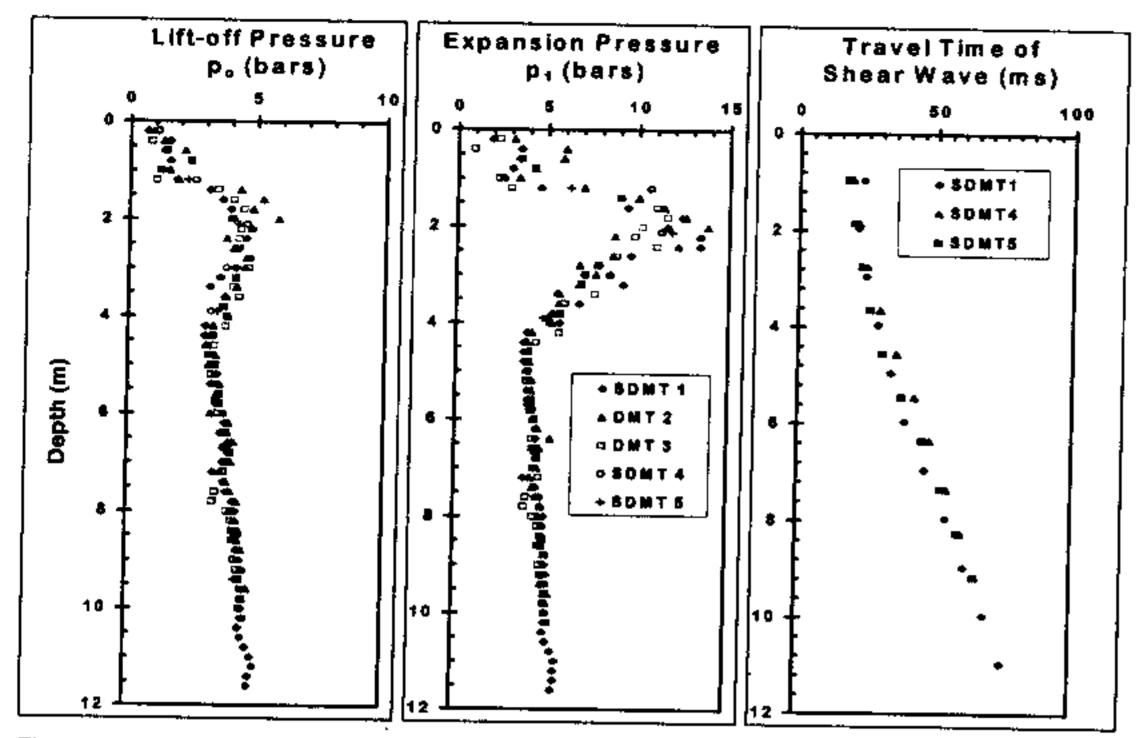


Figure 5. Summary of SDMTs in Amherst clay.

0.9-metre depth intervals). The procedure is similar to the downhole approach used in seismic cone tests. A small OYO Geospace type 14 velocity geophone (model L9) is used as it conveniently fits within the adaptor rod above the dilatometer blade. An HP four-channel digital oscilloscope (Model

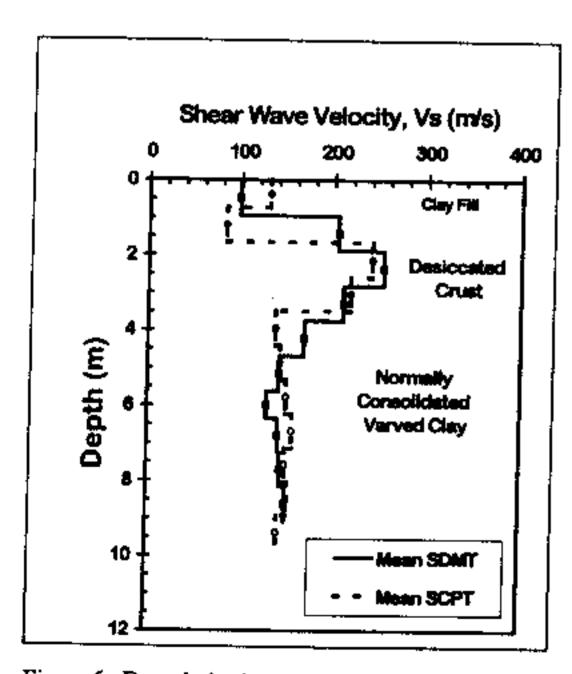


Figure 6. Downhole shear waves at Amherst.

No. 54601A) provides the timing measurements needed for accurate t, arrivals. The results of the SDMT compared well with shear wave values obtained using a commercial Hogentogler seismic cone penetrometer system, as shown by Figure 6.

The SDMTs provided s_u and G₀ values that were used to evaluate the stress-strain-strength behavior of Connecticut Valley varved clay. The predictions are compared with recompression-type direct simple shear (DSS) tests reported by Bonus (1995). These DSS tests were performed on high-quality specimens trimmed from block samples taken at depths ranging from 2.7 m to 8.6 m.

The shear stress (τ) was varied in fractional increments from zero to the maximum shear stress (τ_{max}) equal to the undrained shear strength (s_u). The mobilized shear stress ratios (τ/τ_{max}) were used to generate normalized secant shear moduli, G/G_0 (Eq. 4) and shear strains (Eq. 3). Additional details for various test depths are given by Kates (1996).

An example of the generated stress-strain curve is presented in Figure 7 for Amherst clay at a depth of 6.3 metres. The SDMTs gave $s_u = 21$ kPa and $V_s = 130$ m/s and two DSS tests were available for comparison at this depth (OCR = 2.1). The normalized shear stress ratio (τ/σ_{vo}) versus strain level (γ_s) compares, well with the measured laboratory response. In Figure 8, the normalized shear modulus (G/σ_{vo}) versus strain level (γ_s) is presented for the same depth.

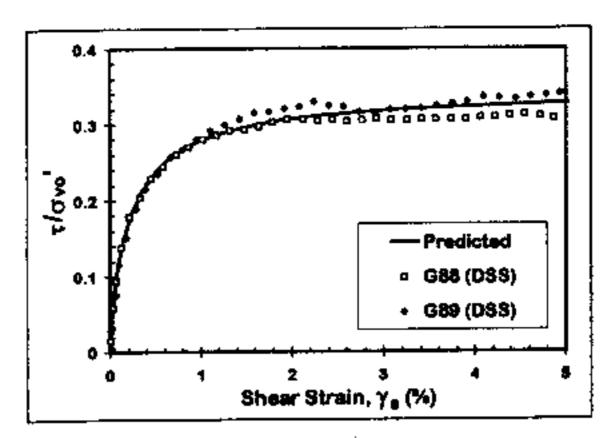


Figure 7. Stress-strain curves for Amherst clay at 6.3 m.

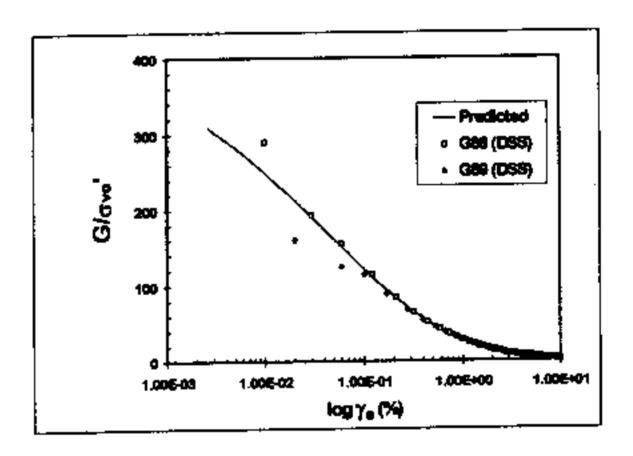


Figure 8. Normalized shear modulus with strain at 6.3 m.

Alternate methods to the modified hyperbola used herein have been suggested for modulus degradation (e.g. Tatsuoka & Shibuya, 1992; LoPresti, et al. 1998) and these may provide superior representation of small- to large strain responses of soils. In fact, a recent logarithmic function suggested by Puzrin & Burland (1998) is capable of perhaps fitting an intermediate value corresponding to the dilatometer modulus (ED) strain level. The intent of the simplified approach discussed herein was to provide a quick means of softening the initial nondestructive Go from seismic data and allow its utilization to stress-strain curve evaluations and direct use in routine analytical solutions, as commonly employed in foundation analysis (Mayne & Dumas, 1997).

6 SPRING VILLA SITE

Seismic dilatometer tests from the Spring Villa NGES in Opelika, Alabama are reported by Martin & Mayne (1998). The site is located within the Piedmont geologic province and consists of over 30 metres of residual silty sands to sandy silts (SM-ML) grading into partially weathered gneiss and decomposed schist (Vinson & Brown, 1997). The soil conditions vary locally, but representative index properties include: liquid limit ≈ 30 to 55, plasticity index \approx NP (nonplastic) to 14, natural water content \approx 25 to 40 percent, fines content (< 75 μ) \approx 45 %, clay fraction (< 2 μ) \approx 10 %, and apparent OCR \approx 2. The groundwater table lies about 3 metres below ground surface.

Three SDMTs were performed at the site using a drill rig to hydraulically advance the blade to the desired depths (Kates, 1996). The contact pressures (p₀), expansion pressures (p₁), and in-situ shear wave velocities (V_s) from two of the SDMTs (AU series) along with one conventional DMT (No. SV99) are shown in Figure 9. The downhole V_s profile derived from the SDMTs compared well with those from standard crosshole tests (CH-SR2) using three cased boreholes at the Spring Villa site.

The SDMT data have been used to develop the stress-strain-strength behavior of Piedmont residual silty sands. The predictions are compared with isotropic triaxial compression tests, reported by Vinson & Brown (1997). The triaxial tests were performed on high quality Shelby tube samples from depths of 4 to 15 metres. Both drained and undrained TX were conducted, with small excess porewater pressures observed in the latter.

A similar analysis procedure was used as described previously, but the SM-ML material was analyzed as a purely frictional material with τ_{max} as expressed in Eq. 5, and the friction angle estimated from the average K_D parameter using Equation 6. Only axial strains are presented in Vinson & Brown (1997), therefore equivalent shear stresses and shear strains were determined from the triaxial compression data (Wood, 1990).

From a series of 23 triaxial tests on the Opelika soils, the effective strength envelope may be represented by an effective stress friction angle of $\phi' = 32^{\circ}$ and effective cohesion intercept c' = 17.4 kPa (Brown & Vinson, 1998). Alternatively, a secant value of $\phi' \approx 37^{\circ}$ to 39° is derived if the material is assumed to be purely frictional (c' = 0). The DMT tests gave a representative $K_D \approx 5$ at these depths, indicating a friction angle of about 37° that was utilized in the modified hyperbola analysis. The shear wave velocity at Opelika is rather constant with depth at $V_s = 200$ m/s for this site within the upper 12 metres.

An example of the generated stress-strain curve is presented in Figure 10 for Piedmont residual silty sand from the Spring Villa NGES at depths of 8 and 10 metres. The shear stress (τ) versus strain level (γ_s) derived from the SDMTs is compared with

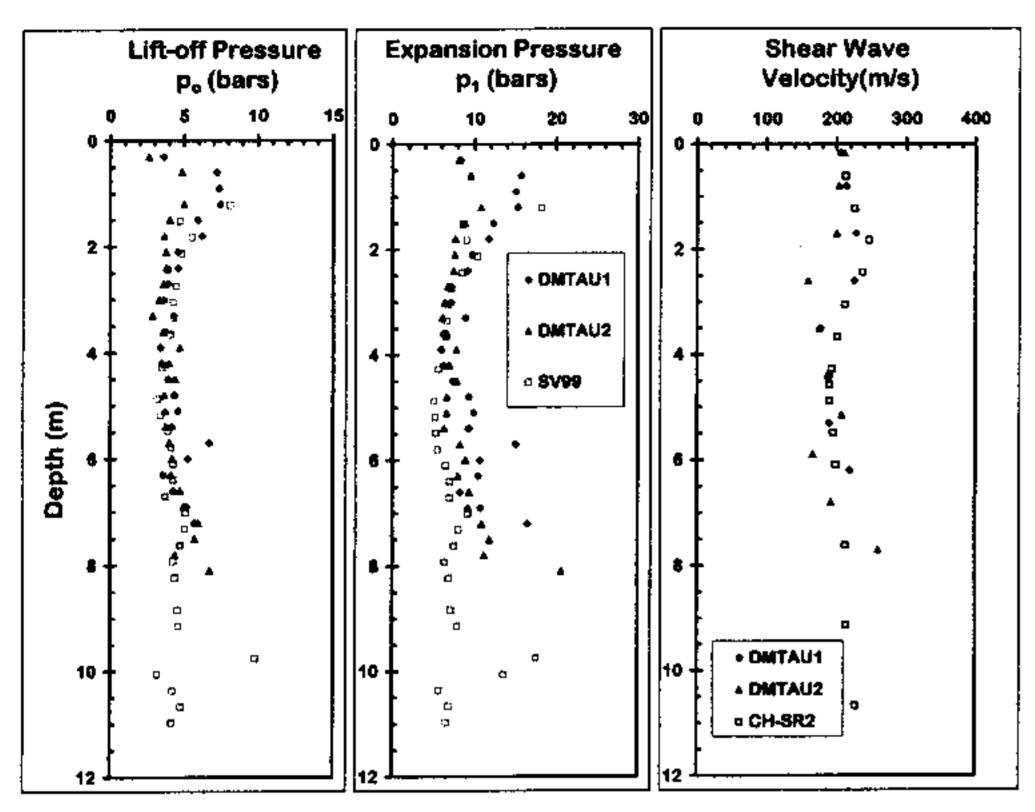


Figure 9. Summary of SDMTs and Crosshole Tests in Piedmont Residual Silty Sand

the laboratory response and seem to slightly underpredict the measured lab behavior. The interpreted shear stresses from the triaxial tests actually gave the parameter, $q = \frac{1}{2}(\sigma_1 - \sigma_3)$. This would yield the maximum value of shear, rather than the secant value associated with an effective stress analysis. The maximum value would be slightly higher than secant values, which partially explains the underprediction of τ by the model.

The aforementioned method may also be applied in a similar manner to results from seismic piezocone penetration tests (SPCPT) for generation of stress-strain-strength relationships (Burns & Mayne, 1996).

As noted previously, the modified hyperbola is merely intended as a simple and expedient means to degrade the initial tangent modulus (G₀) from insitu field geophysical measurements to the appropriate strains at intermediate-levels which correspond to static monotonic foundation and wall loading (Burland, 1989). A number of alternative schemes for nonlinear modulus degradation have been developed that can provide a better fit over the complete small- to intermediate- to large-strain regions of soil behavior (Tatsuoka & Shibuya, 1992), as well as superior models which are based on fundamental principles and rational assessment

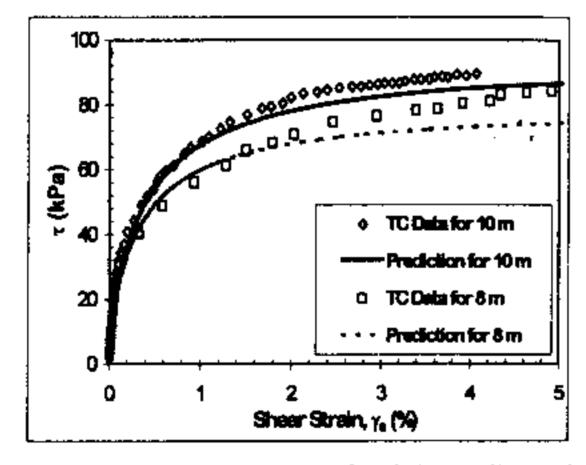


Figure 10. Stress-strain curves for Piedmont silty sand at depths of 8 and 10 metres.

of material properties (e.g., LoPresti et al., 1998; Puzrin & Burland, 1998). The approach adopted here uses an empirical form (Fahey & Carter, 1993) in order to minimize the mathematical complexities and number of required parameters. Moreover, the modified hyperbola permits the quick and direct use of hybrid penetration tests (seismic cone and seismic

flat dilatometer) in generating stress-strain curves and equivalent elastic moduli for use in analytical solutions.

FOUNDATION APPLICATION

The modulus degradation approach can be applied to foundation analysis using the concept of an equivalent elastic modulus (E_s) within a continuum analysis. For shallow foundations under axial compression loading, the magnitude of settlements (δ) can be assessed using displacement influence factors in the general form:

$$\delta = Q I/(B E_s) \tag{9}$$

where B = foundation width, Q is the applied axial load, and I = displacement influence factor that depends on foundation geometry, footing stiffness, roughness, finite layer thickness, and degree of homogeneity (e.g., Poulos & Davis, 1974).

To approximately account for nonlinearity in the load-deflection response, the modified hyperbola can be introduced to give:

$$\delta = Q I/[B E_0\{1 - (Q/Q_u)^{0.3}\}]$$
 (10)

where $E_0 = 2G_0$ (1+0) and Q_u = ultimate axial load from bearing capacity theory. Thus, paired values of Q and δ are obtained as the ratio of (Q/Q_u) is varied from 0 (initial) to 1 (failure). This ratio essentially corresponds to the reciprocal of the overall factor of safety $(Q/Q_u = 1/FS)$.

The Bothkennar experimentation site in Scotland affords an opportunity to evaluate this approach for shallow foundations on soft clay. The results of SDMT in the Bothkennar area have been reported by Hepton (1988) as well as a composite sounding using separate V, and standard DMT pressure data from the actual test site (Nash et al., 1992). Load testing of stacked Kentledge blocks over a square foundation pad with B = 2.2 meters have been presented by Jardine et al. (1995). The undrained bearing capacity can be evaluated as Qu = 6.14 s_u where the undrained strength is assessed per (8). A thin crust overlies the soft clay, thus giving a representative V_s = 92 m/s within the upper 2 to 3 meters. For a rigid square footing on thick soil deposit with homogeneous modulus, the displacement factor I = 0.88.

The measured load-displacement response is compared with the assessment afforded by (10) in Figure 11. There exists uncertainty in the adopted value of Poisson's ratio which changes during loading. In addition, Jardine et al. (1995) suggest that partial drainage may have occurred, as well as complexities due to anistropy and brittleness. Nevertheless, the simplified procedure does in fact

compare well with the test data. Additional case studies have been applied to footings on sand and piles in sand and clay (Mayne & Schneider, 1998).

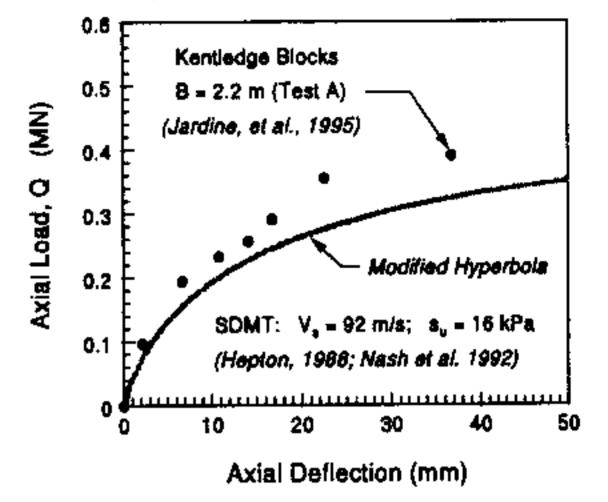


Figure 11. Modulus degradation scheme applied to shallow foundation load test using SDMT data.

8 SUMMARY

A procedure for modulus degradation using a modified hyperbola is discussed for connecting the small-strain nondestructive region to large-strain strengths in order to provide intermediate-level strains. The method utilizes in-situ data obtained from seismic dilatometer tests (SDMT) where downhole shear wave velocities give the initial stiffness (G₀) and inflation pressures (p₀ and p₁) indicate the stratigraphy and soil strength. Applications are presented for evaluating the stress-strain-strength curves for soft varved clay and residual Piedmont silty sands, as well as for assessing foundation deflections using equivalent moduli.

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REFERENCES

Bonus, M.M. 1995. Comparison of recompression and SHANSEP-type DSS stress-strain-strength behaviour of CV varved clay. MS Thesis, Dept. of Civil Eng., Univ. of Massachusetts, Amherst.

- Brown, D.A. & J. Vinson 1998. Comparison of strength & stiffness parameters for a Piedmont residual soil. *Geotechnical Site Characterization*, Vol. 2, Balkema, Rotterdam, 1229-1234.
- Burland, J.B. 1989. Small is beautiful: the stiffness of soils at small strains. Canadian Geotechnical Journal 26 (4), 499-516.
- Burns, S.E. and P.W. Mayne 1996. Small- and high-strain soil properties using the seismic piezocone. *Transportation Research Record* 1548, National Acad. Press, Wash. DC, 81-88.
- Fahey, M. & J.P. Carter 1993. A finite element study of the pressuremeter in sand using a nonlinear elastic plastic model. *Canadian Geotechnical Journal* 30 (2), 348-362.
- Fahey, M., P.K. Robertson, & A.A. Soliman 1994.

 Towards a rational method of predicting settlements of spread footings on sand. Vertical & Horizontal Deformations of Foundations & Embankments (1), ASCE GSP 40, 598-611.
- Hepton, P. 1988. Shear wave velocity measurements during penetration testing. *Penetration Testing in the U.K.*, Thomas Telford, London, 275-278.
- Jardine, R.J., B.M. Lehane, P.R. Smith, and P.A. Gildea 1995. Vertical loading experiments on rigid pad foundations at Bothkennar, Scotland. Geotechnique 45 (4), 573-597.
- Kates (Martin), G.L. 1996. Development and implementation of a seismic flat dilatometer test for small- and high-strain soil properties. *MS Thesis*, Civil Engineering, Georgia Institute of Technology, Atlanta, GA, 173 p.
- Ladd, C.C. 1991. Stability evaluation during staged construction. ASCE Journal of Geotechnical Engineering 117 (4), 537-615.
- LoPresti, D.C.F., O. Pallara, R. Lancellotta, M. Armandi, and R. Maniscalco 1993. Monotonic & cyclic loading behavior of two sands at small strains. ASTM Geotechnical Testing Journal 16 (4), 409-424.
- LoPresti, D.C.F., O. Pallara, V. Fioravante, & M. Jamiolkowski 1998. Assessment of quasi-linear models for sands. *Pre-Failure Deformation of Geomaterials*, Thomas Telford, UK, 363-372.
- Lutenegger, A.J. 1995. Geotechnical behavior of overconsolidated clay crusts. *Transportation Research Record* 1479, National Academy Press, Washington, D.C., 61-74.
- Marchetti, S. 1997. The flat dilatometer: design applications. *Proceedings*, Third Geotechnical Engineering Conf., Cairo Univ., Egypt, 1-25.
- Martin, G.K. & P.W. Mayne 1997. Seismic flat dilatorneter tests in Connecticut Valley varved clay. ASTM Geotechnical Testing Journal 20 (3), 357-361.
- Martin, G.K. & P.W. Mayne 1998. Seismic flat dilatometer tests in Piedmont residual soils. Geotechnical Site Characterization, Vol. 2, Balkerna, Rotterdam, 837-843.

- Mayne, P.W. & R.C. Bachus 1989. Penetration pore pressures in clay by CPTU, DMT, & SBP. Proceedings, 12th Intl. Conf. on Soil Mechanics & Foundation Eng. (1), Rio, 291-294.
- Mayne, P.W. & C. Dumas 1997. Enhanced in-situ geotechnical testing for bridge foundations. Transportation Research Record No. 1569, National Academy Press, Washington, DC, 26-35
- Mayne, P.W. and J.A. Schneider 1998. Characterization of geomaterials by enhanced in-situ testing. *Proceedings*, 27th Annual Midwest Geotechnical Conference, Lansing (Michigan Dept. of Transportation).
- Nash, D.F.T., J.J.M. Powell, and I.M. Lloyd 1992. Initial investigations of the soft clay test site at Bothkennar. *Geotechnique* 42 (2), 163-181.
- Poulos, H.G. and E.H. Davis 1974. Elastic Solutions for Soil & Rock Mechanics, Wiley & Sons, New York, 397 p.
- Puzrin, A.M. and J.B. Burland 1998. Nonlinear model of small-strain behaviour of soils. Geotechnique 48 (2), 217-234.
- Tatsuoka, F. and S. Shibuya 1992. Deformation characteristics of soils and rocks from field and lab tests. Report of the Institute of Industrial Science 37 (1), Serial 235, Univ. Tokyo, 136 p.
- Tatsuoka, F., R.J. Jardine, D. LoPresti, H. DiBenedetto, and T. Kodaka 1997. Characterising the pre-failure deformation properties of geomaterials. *Proceedings*, 14th International Conference on Soil Mechanics & Foundation Engineering, Hamburg, Vol. 4.
- Teachavorasinskun, S., S. Shibuya, & F. Tatsuoka 1991. Stiffness of sands in monotonic & cyclic torsional simple shear. Geotechnical Engineering Congress 1991, Vol. 2, ASCE GSP No. 27, 863-878.
- Vinson, J. L. & D. A. Brown 1997. Site characterization of the Spring Villa geotechnical test site and a comparison of strength and stiffness parameters for a Piedmont residual soil. Report IR-97-04, Highway Research Center, Auburn University, AL, 385 pp.
- Vucetic, M. & R. Dobry 1991. Effect of soil plasticity on cyclic response. ASCE Journal of Geotechnical Engineering 117 (1), 898-107.
- Wood, D.M. 1990. Soil Behaviour and Critical State Soil Mechanics, Cambridge University Press, UK, 462 pp.