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Evaluation of Relative Density and Shear Strength of Sands from CPT and DMT

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Abstract

The paper summarises the experience gained by the writers in the interpretation of the cone penetration test (CPT) and flat dilatometer test (DMT) for the assessment of the geotechnical properties of sands. In the first part of the paper, the problem of determining the relative density (D_R) as function of the penetration test results and ambient stress (σ'), for silica sands, is dealt with. In the second part of the paper, the assessment of the peak angle of shearing resistance (φ'_p) is dealt with. The attention is given to the use of the Bolton's (1986) strength-dilatancy theory in order to estimate φ'_p. Engineering correlations, based on Bolton's (1986) work, are proposed allowing estimation of φ'_p as function of penetration resistance and σ', taking into account the compressibility and the curvilinear shear strength envelope.

Introduction

The concept of relative density (D_R) suggested by Burmister (1948), despite its intrinsic uncertainties and limitations [Tavenas and La Rochelle (1972), Tavenas (1972), Achintya and Tang (1979)], is still extensively used in geotechnical engineering as an index of the mechanical properties of coarse grained soils. Because of the well-known difficulties and the high costs in retrieving good quality undisturbed samples from sand and gravel deposits [Yoshimi et al. (1978), Hatanaka et al. (1988), Goto et al. (1992), Yoshimi (2000)], geotechnical engineers need to

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estimate the in situ D_R using empirical correlations between this parameter and penetration test results. This indirect way of evaluating D_R adds further uncertainties to those already faced when determining the relative density in the laboratory. The first attempt to correlate the blow-count of the Standard Penetration Tests (N_{SPT}) to the density of sands is linked to the works by Terzaghi and Peck (1948) and Gibbs and Holtz (1957). The continuous interest in this kind of correlation is testified by the more recent works by Skempton (1986) and Cubrinovski and Ishihara (1999). As far as the CPT is concerned, a pioneering work can be dated back to Schmertmann (1976). He presented the first comprehensive correlation between the cone resistance (q_c) and D_R on the basis of static Cone Penetration Tests (CPT) performed in Calibration Chambers (CC). Such correlation relates D_R to the effective overburden stress (σ'_{vo}) and is applicable to normally consolidated (NC) fine to medium unaged sands. Twenty five years later, based on the results of 484 CC-CPT's performed in three silica sands, the writers have attempted to present similar correlations considering the effect of CC size on the measured q_c and giving appropriate consideration to mechanically overconsolidated (OC) sands. The assessment of D_R represents the most common intermediate step in estimating the stress-strain-strength characteristics of sands and gravels.

In the second part of the paper, we will deal with the assessment of the peak friction angle φ'_p of sands, making reference to the simplified Bolton's (1986) strength-dilatancy theory. Based on a theoretically sound framework of Rowe (1962), the input parameters required to estimate φ'_p are: D_R, the friction angle φ'_{cv} at critical state and a parameter Q related to sand compressibility. The presentation also includes a comprehensive discussion about the intrinsic parameters φ'_{cv}, Q.

Evaluation of relative density

The first attempt to correlate the penetration resistance of the cone penetration tests (q_c) to the density of sands dates back to the work by Schmertmann (1976). He presented the first comprehensive correlation between q_c and relative density (D_R) on the basis of CPTs performed in the Calibration Chamber (CC). Such a correlation is applicable to normally consolidated (NC) fine to medium, unaged, clean sands. Schmertmann (1976) suggested a correlation between the cone resistance (q_c) the relative density and the vertical effective stress (σ'_{vo}), using the results of CPT's performed on sands in the CC of the University of Florida. The analytical expression takes the form:

$$q_c = C_o \cdot \left(\sigma'_{vo}\right)^{C_1} \cdot \exp(C_2 \cdot D_R) \tag{1}$$

$$D_R = \frac{1}{C_2} \ln\left[\frac{q_c \cdot \left(\sigma'_{vo}\right)^{C_1}}{C_o}\right] \tag{2}$$

where: C_o, C_1, C_2 = empirical correlation factors

Since the pioneering work by Schmertmann, many CC's have been put into operation in North America, Europe Australia and Japan generating a large data-base of CPTs performed in different sands and providing a deeper insight into the merits and limitations of this kind of large-scale laboratory test and of the empirical correlations that can be obtained. The key points that have emerged from these experiments can be summarised as follows:

- The analysis of the variance (Tumay 1976), performed to investigate the relative importance of the different factors influencing the magnitude of the q_c of silica sands measured in CC tests, led to the conclusion that the relative density (D_R) and the consolidation stress tensor (i.e. the level of effective stress existing in the specimen, prior penetration) are the most important variables that influence q_c. (Harman 1976, Schmertmann 1976, Garizio 1997).
- The correlation of q_c vs. D_R and σ'_{vo} holds only for NC sands. A correlation for NC and OC deposits should refer to the effective mean in situ stress σ'_{mo} instead of σ'_{vo}.
- Stress and strain history that can be reproduced in the laboratory play a secondary role with the exception of increase of the horizontal effective consolidation stress as result of the mechanical overconsolidation which concurs to the value of the relevant stress tensor (Jamiolkowski et al. 1988).
- In the case of siliceous sands, their grain shape and crushability play a secondary role (Robertson & Campanella 1983, Lunne et al. 1997). The influence of grading on the penetration resistance has not been systematically investigated. However, the use of the correlations obtained from CC experiments leads to underestimation of D_R in the case of sand deposits containing more than 5 to 10 % of fines (Jamiolkowski et al. 1988)
- Thanks to the works by Dussealt and Morgenstern (1979) and Barton and Palmer (1989) which have investigated the effect of geological time on porosity, fabric and mechanical properties of coarse grained soil deposits, it is obvious that the empirical correlations based on the results of tests performed on laboratory reconstituted specimens, are applicable only in the case of young, unaged NC soils. Skempton (1986) has shown a certain influence of aging on the correlations between D_R and the blow-count of the Standard Penetration Test (N_{SPT}). Analogously, it is reasonable to suppose that aging influences the D_R vs. q_c correlations (see also Wride et al. 2000).
- Due to the finite dimensions of the CC, the measured cone resistance is affected by an error in comparison to that obtainable in the case of an infinite sand deposit with the same relative density. This phenomenon, named chamber size effect (Schmertmann 1976, Parkin and Lunne 1982, Baldi et al. 1986, Foray 1986, Mayne and Kulhawy 1991, Tanizawa 1992, Salgado 1993) leads, within some boundary conditions, to an underestimate or overestimate of the field q_c depending on the boundary conditions imposed on the CC specimen during the cone penetration. The magnitude of such an underestimation or overestimation depends on the crushability and compressibility of the test sand, the ratio of the CC specimen diameter (D_c)

to that of the cone (d_c), D_R and confining stresses applied to the CC specimen.

- The degree of saturation and boundary conditions imposed on the CC specimen during the cone penetration are much less influential on the q_c.

The previously illustrated considerations also apply to the penetration resistance (q_D) as obtained from blade thrust readings in the dilatometer tests (DMT). It is worthwhile to point out that this kind of measurement is not routinely performed in dilatometer tests (Marchetti 1980, 1997).

In light of what has been stated above, the writers propose empirical correlations, similar to that used by Schmertmann (1976) and based on the results of 484 CC-CPT's and 136 CC-DMT's that have been performed in three silica sands. As far as DMT's are concerned, correlations between the lateral stress index K_D and D_R are also shown. The practical use of the proposed correlations is the prediction of the relative density of granular deposits. This task can be accomplished by using the proposed correlations and keeping in mind the intrinsic limitations of such correlations as already discussed.

Experimental Data. The CPTs and DMTs have been performed in Calibration Chambers of ENEL of Milan and the research institute ISMES of Bergamo. The apparatus houses 1.2 m in diameter and 1.5 m in height specimen reconstituted by means of pluvial deposition in air (Bellotti et al. 1982, 1988, Garizio 1997, Felice 1997). After deposition, samples were subject to the one-dimensional compression in order to apply the desired consolidation stress level and stress-history. After the consolidation stage, the penetration test (CPT or DMT) was performed, applying to the CC specimen one of the four available boundary conditions (BC).
Most of the tests were performed under two BC's:

- BC-1: constant axial (σ'_a) and radial (σ'_r) effective stresses;
- BC-3: constant axial effective stresses (σ'_a) and zero radial strain (ε_r);

In addition, a limited number of CC tests were carried out using either BC-2 (axial strain ε_a=0, σ'_r = constant), or BC-4 (ε_a = ε_r = 0) during the penetration stage. Table 1 indicates the percentage of tests performed under each BC's.

All CPT's were performed using the cylindrical Fugro-type electrical cone tips. In most tests, the standard cone tip 35.6 mm in diameter (Lunne et al. 1997) has been employed. A limited number of tests were also performed using cone tips

Table 1. Percentage of CPT's and DMT's performed under different boundary conditions

Test	BC-1 (%)	BC-2 (%)	BC-3 (%)	BC-4 (%)
CPT's (total 484 tests)	66	11	20	3
DMT's (total 136 tests)	86	0	11	1

having diameters (d_c) equal to 25.4, 20, 11 and 10 mm. These tests were aimed at investigating the influence of the CC diameter (D_c) to d_c ratio (R_d) on the q_c measured under different BC's.

Most of the DMTs were performed using a standard dilatometer (Marchetti 1980). The probe is 14 mm thick, 95 mm wide and 220 mm high. An expandable steel membrane, 60 mm in diameter, is located on one side of the probe; a load cell for the measurement of the penetration resistance (q_D) is located just above the probe. Few tests were performed using a research dilatometer (RDMT) (Fretti et al. 1992, 1996, Bellotti et al. 1997). The main differences between standard dilatometer and RDMT are: i) the expandable steel membrane of RDMT is equipped with strain gauges, so that it is possible to monitor the complete expansion curve, ii) the structure of the RDMT probe is much stiffer in comparison of the standard DMT, even though the dimensions of the two probes are identical.

The CC tests were carried out in three well-known silica sands: fine to medium Ticino (TS), Toyoura (TOS) and Hokksund (HS) sands. The index properties of these sands are reported in Table 2 and Figure 1.

Data Interpretation

Size effect. The tests run with different cone sizes confirmed the well-known fact that the penetration resistance measured in the CC is influenced by the imposed

Table 2. Index Properties of Test Sands

PARAMETER		TICINO	HOKKSUND	TOYOURA
γ_{max}	[kN/m^3]	16.67	17.24	16.13
γ_{min}	[kN/m^3]	13.64	14.10	13.09
G_S	[-]	2.68	2.72	2.65
U_C	[-]	1.30	1.91	1.31
D_{50}	[mm]	0.60	0.45	0.22
Quartz	[%]	30	35	90
Feldspar	[%]	65	55	8
Mica	[%]	~5	~10	~3
ϕ'_{cv}	[°]	33	34	32

γ_{max} and γ_{min} = maximum and minimum dry density respectively
G_S = specific density
U_C = uniformity coefficient
D_{50} = mean grain size
ϕ'_{cv} = friction angle at critical state

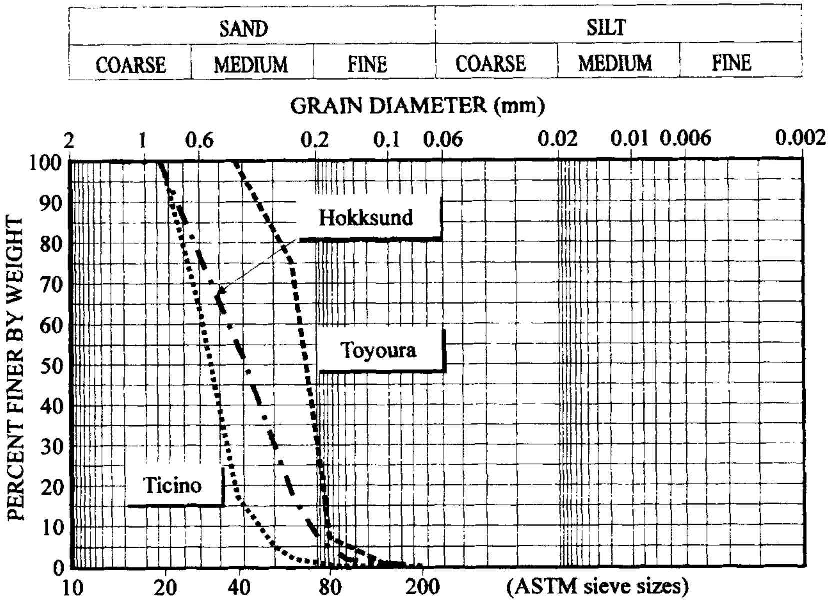

Figure 1. Grain size distribution of three test sands

BC's. Such effect is inversely proportional to the R_d and decreases with increasing the sand compressibility. Further details on such effect can be found in the already mentioned works by Schmertmann (1976), Parkin and Lunne (1982), Baldi et al. (1986), Foray (1986), Mayne and Kulhawy (1991), Tanizawa (1992), Salgado (1993), Salgado et al. (1998). The measured penetration resistance appears to be independent on the BC's when the R_d is sufficiently large, i.e. 70 to 100 for silica sands of moderate to low compressibility respectively. Under these conditions the penetration resistance measured in the CC matches the field value. The measured penetration resistance were therefore corrected for chamber size effect by means of the following empirical equation (Tanizawa 1992, Garizio 1997, Felice 1997):

$$CF = a\left[(D_R)^b\right]^m \quad [-] \tag{3}$$

where:

CF = correction factor by which the measured penetration resistance has to be multiplied

a, b = empirical coefficients function of R_d inferred from the CC performed in TS and TOS using CPT tips having different size

m = + 1 and –1 for BC-1 and BC-3 respectively.

The values of coefficients a and b for different R_d ratios are given in Table 3. The lower bound of $D_R = (D_R)_{min}$ below which CF=1 is also reported in Table 3. The trend of the empirical CF, yielded by Eq. 3, vs. R_d is similar, although generally lower, to what achieved by Salgado (1993) from numerical modelling.

The measured q_D have not been corrected to account for CC size effect. Based on limited CC evidence, it appears that there is no need to correct the penetration resistance of a plate blade because the experimentally determined q_D is not influenced by the finite dimensions of the chamber (Felice 1997), even though the reasons are not well understood.

Table 3. Coefficient a and b of Eq. 3

$R_d = \frac{D_c}{d_c}$	A	B	$(D_R)_{min}$ %
100	0	0	100
60	0.412	0.221	55.8
47.2	0.166	0.457	50.8
33.6	0.090	0.624	47.4
22.1(4)	0.054	0.827	34.1
$(D_R)_{min} = D_R$ value in percent at which CF should be taken equal to one			

Proposed correlations. The writers adopted the following equations to fit the experimental data:

1) the same equation used by Schmertmann (1976)

$$q_c = C_0 p_a \left(\frac{\sigma'}{p_a} \right)^{C_1} \exp(C_2 D_R) \tag{4}$$

from which is it possible to obtain:

$$D_R = \frac{1}{C_2} \ln \left[\frac{q_c / p_a}{C_0 (\sigma' / p_a)^{C_1}} \right] \tag{5}$$

where:

q_c = measured cone resistance multiplied by CF of Eq. 3

σ' = an initial effective geostatic stress component or stress invariant $\lfloor FL^{-2} \rfloor$

D_R = relative density (as decimal)

(4) tests performed in a smaller CC in Japan by Tanizawa (1992) in TOS

C_o, C_1, C_2 = non dimensional empirical correlation factors, see Table 4 for CPT's
p_a = atmospheric pressure expressed in the same unit system of stress and penetration resistance (i.e. 98.1 kPa or 1 bar etc.)

2) the equation proposed by Lancellotta (1983)

$$D_R = A_o + B_o \cdot X \quad (6)$$

where:

$$X = \ln\left[\frac{q_c}{(\sigma'_{vo})^{\alpha}}\right]$$

A_o, B_o and α = empirical correlation factors (see Table 5). The parameter α is obtained following an optimisation process which minimises the differences between computed and measured values of the penetration resistance in terms of standard deviation. In this case q_c and σ'_{vo} are in kPa.

The same equations have been used in the case of DMT's. The empirical correlation factors obtained from DMT results are reported in Tables 5 & 6.

As to the definition of the effective stress σ' to be introduced into Eqs. 4 and 5, the following should be taken into account:

- Zolkov and Weisman (1965) postulated that N_{SPT} is controlled by the horizontal in situ effective stress (σ'_{ho}).
- Similar experimental evidence emerged from CPT's performed in CC's (Schmertmann 1971, 1972, Baldi et al. 1986, Houlsby and Hitchman 1988, Mayne and Kulhawy 1991, Salgado 1993, Garizio 1997, Felice 1997). It shows that the magnitude of the penetration resistance is much more influenced by σ'_r than by σ'_a.
- The above statement suggests that any rational correlation between penetration resistance and relative density should be related to the mean (σ'_{mo}) or horizontal (σ'_{ho}) effective geostatic stresses rather than to the (σ'_{vo}). The lesson learned is that the correlation between the penetration resistance and relative density involving σ'_{vo} is applicable only to NC deposits of coarse grained soils in which K_o ranges from 0.4 to 0.5 remaining more or less constant with depth.

Given the above considerations, Figures 2 and 3 report the $D_R = f(q_c, \sigma')$ correlations for NC (Fig. 2) and (NC+OC) (Fig. 3) dry silica sands respectively. As to the latter, the writers fully appreciate the extreme difficulties linked with the estimation of σ'_{ho} in sand and gravel deposits. Overall, but particularly for coarse-grained soils, the determination of σ'_{ho} or of the coefficient of the earth pressure at rest (K_o) in situ is still an unsolved problem in geotechnical engineering.

Table 4. Coefficients C_o, C_1 and C_2 of Eqs. 4 and 5 (CPT's)

$$q_c = C_o p_a \left(\frac{\sigma'}{p_a}\right)^{C_1} \exp(C_2 D_R) \qquad D_R = \frac{1}{C_2} \ln\left[\frac{q_c / p_a}{C_o\left(\sigma' / p_a\right)^{C_1}}\right]$$

$\sigma' = \sigma'_{vo}$	TS	TS + TOS + HS
C_o	17.74	17.68
C_1	0.55	0.50
C_2	2.90	3.10
R	0.90	0.89
$\bar{\sigma}$	0.12	0.10
N	305	180
$\sigma' = \sigma'_{mo}$	TS	TS + TOS + HS
C_o	23.19	24.94
C_1	0.56	0.46
C_2	2.97	2.96
R	0.87	0.87
$\bar{\sigma}$	0.10	0.10
N	299	484

R = correlation coefficient
$\bar{\sigma}$ = standard error
N = number of CC tests considered

Table 5. Coefficients A,B and α of eq. 6 (CPT's & DMT's)

$$D_R = A_o + B_o \cdot X \qquad X = \ln\left[\frac{q_c}{(\sigma'_{vo})^{\alpha}}\right]$$

CC test	A_o	B_o	α	R	$\bar{\sigma}$	N
CPT's	-1.292	0.268	0.52	0.94	7.9	456
DMT's	-1.082	0.204	0.36	0.92	6.6	100

R= Correlation coefficient
$\bar{\sigma}$ = standard error
N = number of CC test considered

Table 6. Coefficients C_o, C_1 and C_2 of Eqs. 4 and 5 (DMT's)

$$q_c = C_o p_a \left(\frac{\sigma'}{p_a}\right)^{C_1} \exp(C_2 D_R) \qquad D_R = \frac{1}{C_2}\ln\left[\frac{q_c / p_a}{C_o\left(\sigma' / p_a\right)^{C_1}}\right]$$

$\sigma' = \sigma'_{vo}$	TS	TS + TOS + HS
C_o	19.14	20.64
C_1	0.62	0.52
C_2	3.61	3.71
R	0.88	0.88
$\bar{\sigma}$	0.11	0.10
N	57	69
$\sigma' = \sigma'_{mo}$	TS	TS + TOS + HS
C_o	26.99	26.62
C_1	0.60	0.49
C_2	3.75	3.80
R	0.91	0.89
$\bar{\sigma}$	0.12	0.11
N	110	136

R = correlation coefficient
$\bar{\sigma}$ = standard error
N = number of CC tests considered

Nevertheless, the estimate of σ'_{mo} in coarse-grained soils is facilitated by the following considerations:

- In NC deposits the upper limit of K_o can be taken as $1 - \sin\varphi'_{cv}$ (i.e. the angle of shearing resistance at critical state).
- In heavily OC sands (i.e. OCR = 15) K_o is not greater than 1.0 as suggested by the CC results (Jamiolkowski et al. 1988).

Figures 4 & 5 show Eq. 6 in the case of CPT's and DMT's respectively. Figures 4 & 5 enable one to appreciate the accuracy of Eq. 6 to fit the experimental data that are also plotted in the Figures together with the limits corresponding to $\pm 2\bar{\sigma}$.

Empirical correlations were also established between the lateral stress index (K_D) and D_R. K_D is computed from DMT results in the following way:

$$K_D = \frac{p_o - u_o}{\sigma'_{vo}} \tag{7}$$

where:

p_o = lift-off pressure

u_o = pore pressure, prior to penetration and expansion

σ'_{vo} = vertical effective stress, prior to penetration and expansion

The dependence of K_D on D_R is clearly shown in Figure 6 for the three considered sands. Two different equations were used to fit the experimental data:

$$K_D = C_o(\sigma')^{C_1} p_a^{1-C_1} \exp(C_2 D_R) \tag{8}$$

$$K_D = A\exp(BD_R) \tag{9}$$

Eq. 8 is similar to Eq. 4 with the only difference that, instead of considering the penetration resistance, the lateral stress index appears in the formula. Eq. 9 is much more crude and does not take into account the stress level prior to penetration and expansion (σ'). Moreover, D_R is expressed as a fraction of one.

The accuracy of Eq. 8 is not influenced by the choice of the effective stress.

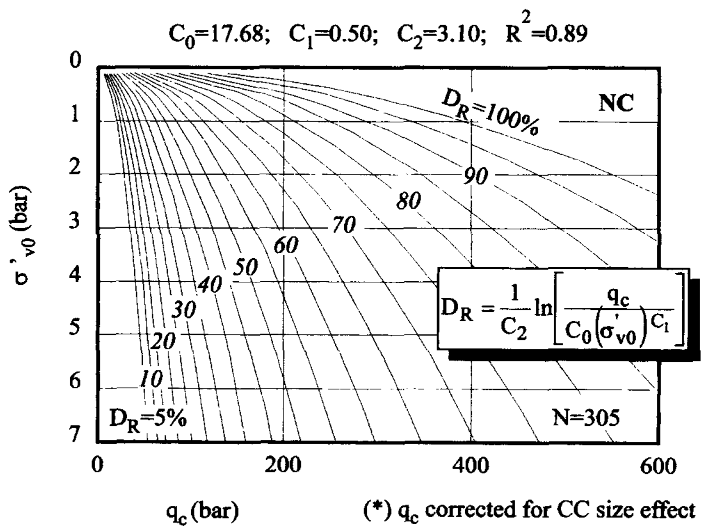

Figure 2. Relative density of NC siliceous sands

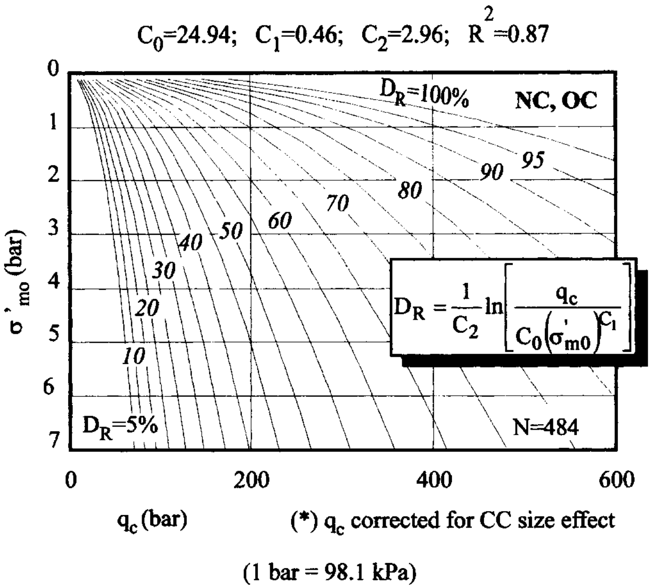

Figure 3. Relative density of NC and OC siliceous sands

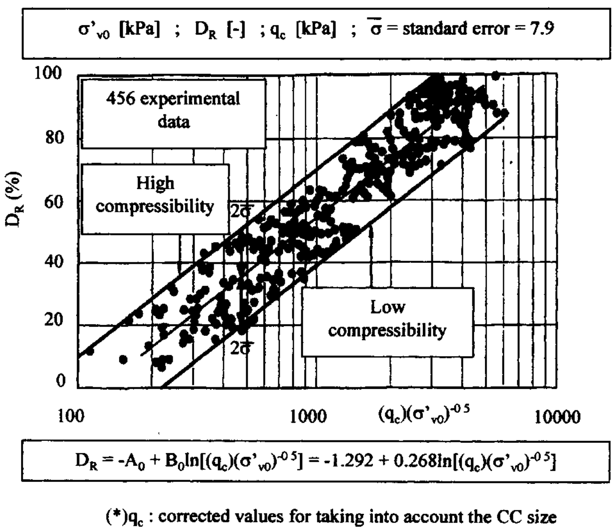

Figure 4. Experimental correlation D_R-q_c-σ'_{v0} for mainly NC sands of different compressibility (Lancellotta, 1983; Garizio, 1997)

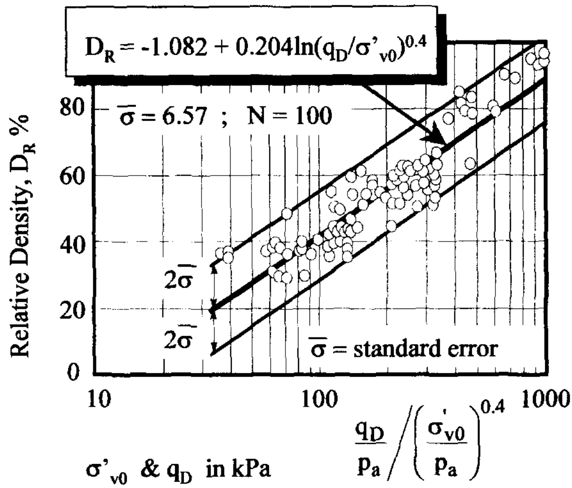

Figure 5. Relation between wedge resistance and relative density – Calibration chamber tests in Ticino sands (Felice, 1997)

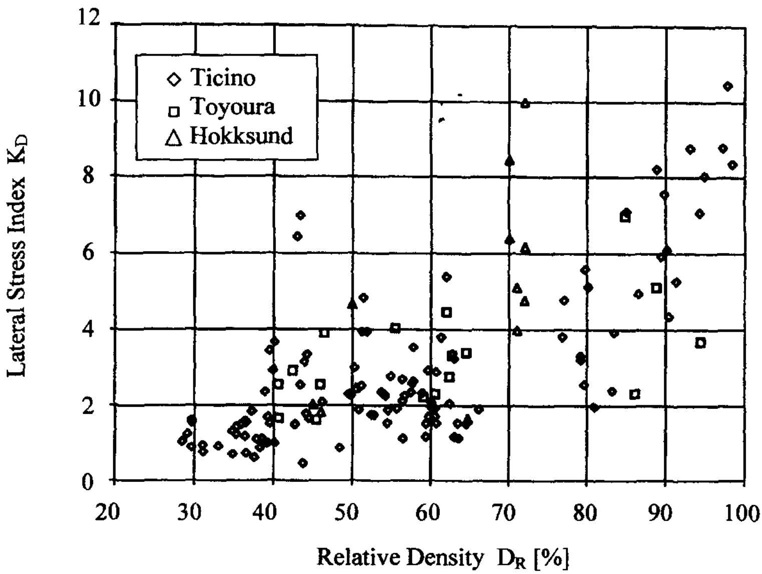

Figure 6. K_D vs D_R from CC-DMT's in three siliceous sands

In Table 7 the correlation coefficients that have been obtained for the case $\sigma' = \sigma'_{vo}$ (NC tests) are reported. It is worthwhile to point out that the stress exponent assumes negative values, irrespective of the selected effective stress. It is also important to notice that the use of a simpler formula. like Eq. 9 does not involve relevant reduction in accuracy. In any case, the correlations between K_D and D_R are less accurate than those between q_c (or q_D) and D_R. Figure 7 shows Eq. 9 $\pm 2\bar{\sigma}$ and the experimental data. The empirical correlation factors used in Figure 7 are slightly different from those reported in Table 7 because they have been obtained disregarding those test results with a deviation from the computed value higher than $\pm 2\bar{\sigma}$.

Degree of saturation. All CPT's and DMT's used to derive the previously shown correlations were carried out on dry specimens. Only a limited number of tests were performed in saturated TS (Bellotti et al. 1988) showing little influence of the saturation on the measured penetration resistance. Strictly speaking, these correlations are applicable to dry fine to medium clean, unaged, uncemented silica sands of low to moderate compressibility in which the static cone penetration process corresponds essentially to a drained process.

In order to overcome, at least partially, the limitations derived from the above specified condition the following indications can be helpful:

Table 7. Coefficients C_0, C_1, C_2 A,B eqs.8 and 9 (DMT's)

$$K_D = C_0 (\sigma')^{C_1} p_a^{1-C_1} \exp(C_2 D_R) \qquad K_D = A \exp(B D_R)$$

Eq. 8 $\sigma' = \sigma'_{vo}$	TS	TS+TOS+HS
C_0	$5.3 \cdot 10^{-3}$	$6.6 \cdot 10^{-3}$
C_1	-0.18	-0.25
C_2	2.60	2.29
R	0.78	0.76
$\bar{\sigma}$	12	12
N	58	73
Eq. 9	NC	NC+OC
A	0.53	0.57
B	2.42	2.56
R	0.71	0.71
$\bar{\sigma}$	13	13
N	73	136

R= Correlation coefficient
$\bar{\sigma}$ = standard error
N = number of CC test considered

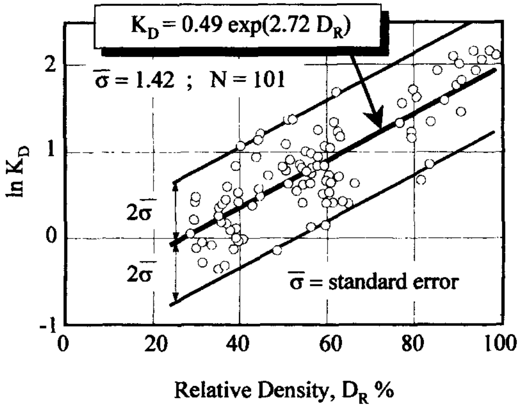

Figure 7. Relation between lateral stress index and relative density – Calibration chamber tests in Ticino sands (Felice, 1997)

- The comparison between q_c(dry) and q_c(saturated) resulting from CC-CPT's carried out on almost identical CC specimens of TS shows very small differences. Similar conclusions have been reached by Schmertmann (1976) comparing CC tests performed on dry and nearly saturated quartz Ottawa sand. It is worthwhile to point out that the use of the previously shown correlations in saturated sands leads to an underestimation of D_R, whose magnitude can be inferred from the following empirical relationship:

$$\frac{D_R(\text{saturated}) - D_R(\text{dry})}{D_R(\text{dry})} 100 = -1.87 + 2.32 \ln \frac{q_c}{\left(\sigma'_{vo} p_a\right)^{0.5}} \tag{10}$$

The above exposed formula becomes meaningless for $q_c / (\sigma'_{vo} p_a)^{0.5} \leq 2.24$; underestimation of D_R, for the sands considered in this paper, ranges between 7 and 10 %.

Aging and Cementation. As far as the effects of cementation and ageing on the penetration resistance are concerned, currently there is a lack of information able to estimate and quantify their influence. Schmertmann (1991) has shown that the accumulation of secondary compression in sands tends to moderately increase the cone resistance, see for example Kulhawy and Mayne (1990).

More relevant might be the impact of even light cementation on q_c (Puppala 1993, Puppala et al. 1995, Eslaamizzad 1997). It may be useful to mention that the use of correlations similar to those here presented and established on freshly deposited sands, in aged and/or cemented deposits leads to an overestimation of the relative density.

Evaluation of shear strength

When dealing with the shear strength of non-cemented granular materials, the friction angle, resulting from the secant slope of the failure envelope, is, in general, the reference parameter
for both the simplified design approaches (e.g. limit equilibrium and limit analysis methods) and the most complex multi-surface non-linear elasto-plastic work-hardening models.

The appropriate definition of the peak friction angle φ_p and the operational friction angle φ_{op}[(5)] referring to the simplified design approaches, appears to be even more difficult than when more complex models of soil behaviour are used. As a matter of fact, the operational value of φ_{op}, for a given boundary value problem, is a function, among the other state parameters, of all the components of stress and strain tensors which can be reliably assessed only through sophisticated theoretical approaches. On the other hand, using simplified design methods, the most appropriate operational value of φ_{op}, should be theoretically evaluated with reference to the average values of the significant state parameters within the yielding volume of soil. This kind of evaluation, using simplified design methods, is only possible in an approximate manner and for a limited number of the recurrent boundary values problems in Soil Mechanics.

In light of the previous considerations, in the following part of the paper, the basic principles governing the shear strength of sandy soils, with particular reference to the framework and the relationships proposed by Bolton (1986), are illustrated and partially worked out to widen the possible practical applications when simplified models of soil behaviour are adopted. In particular, an evaluation procedure to estimate the operational friction angle from CPT is illustrated. It allows the evaluation of operational friction angles of sands having different mineralogical composition and/or grain size distribution, once the point resistance q_c and the mean geostatic effective stress σ'_{mo} are known.

Basic principles governing shear strength of sandy soils. The basic principles that govern the shear strength of granular materials (i.e. the critical state concept, the energy dissipation by particle rearrangement, the dilatancy and the dependency of the latter on the current state parameters) and the influence of secondary factors such as strain conditions and anisotropy, have already been clearly pointed out by Casagrande (1936), Taylor (1948), Rowe (1962), Schofield & Wroth (1968), Bolton (1986), Mitchell (1976), Lade & Lee (1976), Ladd et al. (1977) and Tatsuoka et al. (1986). Looking at the framework and the related equations proposed by the aforementioned researchers, the main components of shear strength of sands can be split as follows:

- The pure frictional resistance between smooth surfaces of the sand mineral, quantified by the interparticle friction angle φ_μ;
- The particle rearrangement component determining the strength increase from φ_μ to the constant volume friction angle φ_{cv};

(5) Average mobilized friction angle in correspondence of the general failure for current geotechnical boundary value problems.

The dilation component determining the difference between the peak strength (represented by the peak friction angle φ_p) and the steady state strength corresponding to φ_{cv}.

Based on the results of triaxial (TX) and plane strain (PS) compression tests obtained for different sands, Bolton (1986) attempted an empirical correlation to assess the peak friction angle that takes into account the relative density (D_R), the mean effective stress at failure σ'_{mf}, and the sand type in terms of grain size distribution, mineralogy and grain shape.

The equation originally proposed by Bolton (1986) can be written as follows:

$$\varphi_p - \varphi_{cv} = m\{D_R[Q - \ell n(\sigma'_{mf})] - R\} \qquad \varphi_p \geq \varphi_{cv} \tag{11}$$

where: φ_p = peak friction angle in [°]; φ_{cv} = constant volume friction angle [°]; m = coefficient equal to 3 or 5 for axisymmetric (TX) and plane strain (PS) conditions respectively; D_R = relative density; Q = particle strength parameter (reported in Table 8); σ'_{mf} = mean effective stress at failure in [kPa]; R = coefficient that in a first approximation is a function of (φ_{cv}-φ_μ) and normally it is assumed equal to 1 for sands; φ_μ = pure friction angle between smooth surfaces of the mineral forming the considered sand.

Table 8. Q values suggested by Bolton (1986)

GRAIN MINERAL	Q
Quartz and feldspar	10
Limestone	8
Anthracite	7
Chalk	5.5

Looking in detail at Eq. 11 it can be remarked (see also Fig. 8) that it represents a bunch of straight-lines in the plane φ_p; $\ln(\sigma'_{mf})$ converging in the point with coordinates $\ln(\sigma'_{mf})$=Q and φ_p= (φ_{cv}-m·R), moreover, the angular coefficient of the straight lines is equal to (m·D_R).

The common point of the aforementioned straight-lines is quite peculiar since its coordinates express some intrinsic features of the considered sand not depending on the state parameters (e.g. the typical combinations of D_R and σ'_{mf}).

In first approximation, the term (m·R) can represent the shear strength component due to the grains rearrangement at constant volume strains (Rowe, 1962), so it can be expressed by the difference between φ_{cv} and φ_μ.

The term Q can be expressed by the logarithmic function of an appropriate equivalent grain yield stress (σ'_c) that can be related to the grain crushing strength for coarse materials (p'_{fm}) as definied by Biliam (1967) and Marsal (1967) or can be

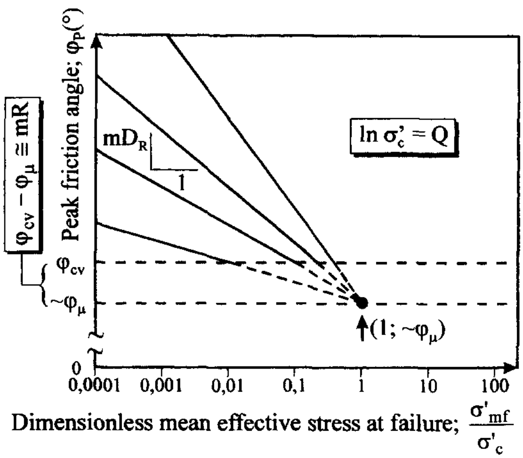

Figure 8. Re-plotting of Bolton's (1986) relationships referring to pure friction angle (φ_{μ}) and dimensionless confining stress

referred to the threshold confining stress level at which, for a given sand, a single value of void ratio is obtained independently from its initial relative density, fabric and arrangement of the solid skeleton (see Figures 9 and 10).

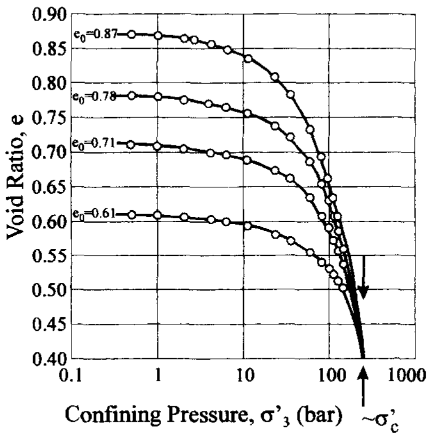

Figure 9. Isotropic compression tests on Sacramento river Sand (from Lee and Seed, 1967)

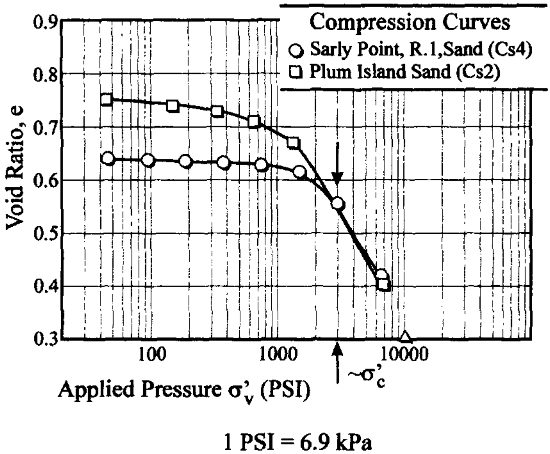

Figure 10. K_0-compression tests for two similar sands (from Roberts, 1964)

In order to justify this simple model from a mechanical point of view it is possible to imagine that, when the isotropic stress level is able to destroy the internal strength of the sand grains, the shear stress increments can be only sustained by the pure friction mobilized between smooth surfaces of the mineral forming the grains, then without any contribution from the particle rearrangement and, of course, from the dilatancy.

Based on the above considerations, Eq. 11 can be rewritten, referring explicitly to the intrinsic and state parameters influencing the peak friction angle φ_p (see also Fig. 8):

$$\varphi_p - \varphi_\mu = m \cdot D_R \cdot \ell n(\sigma'_c / \sigma'_{mf}) \qquad \varphi_p \geq \varphi_\mu + m \tag{12}$$

With reference to the silica sands considered by Bolton (1986), in order to obtain the same results of Eq. 11, it is necessary to introduce the following parameters into Eq. 12:

$\sigma'_c = 22026.5$ (kPa);

$\varphi_{cv}-\varphi_\mu = 3°$ for axisymmetric conditions (TX) and

$\varphi_{cv}-\varphi_\mu = 5°$ for plane strain conditions (PS).

As to the value of σ'_c, it is in good agreement with the expected values observed and/or extrapolated from the isotropic compression tests of Lee & Seed (1967) and Robertson (1964) reported in Figs. 9 and 10.

As far as the constant volume friction angle φ_{cv} is concerned, the reinterpretation of the fitting parameters by Bolton (1986), under the light of the proposed approach (Eq. 12), points out possible different values from TX and PS conditions.

Although many authors, e.g.: Hanna et al. (1987); Schanz (1998), have produced the experimental evidence that φ_{cv}(TX) and φ_{cv} (PS) are the same, the uncertainties linked to the large strain and strain non-uniformities still leave some open questions with respect to this problem..

Apart the practical problems of test equipment, other sources of uncertainties, that can influence the φ_{cv} values, might also arise from the fitting procedures of the experimental data carried out by Bolton (1986). Therefore, considering all the above aspects, the difference of 2° between φ_{cv} values from TX and PS conditions could be easily justified and accepted, also considering the empirical nature of Eq. 12.

In order to validate the proposed modification to the original Bolton formula, it can be interesting to note that Eq. 12 is very similar to the one proposed by Barton (1973) to describe the curved shear strength envelope of rock joints. Moreover, the equation by Baligh (1975, 1976), describing the curvature of the sands failure envelope at a given relative density, can also be re-written in the form of the above equations.

Of course the proposed simplification of the Bolton's (1986) formula, based on the use of the intrinsic parameters (φ_{μ}, σ'_c) characterizing the coarse granular media behaviour, must be validated by further experimental data. Nevertheless, it can be used as a reference framework for analysing the basic contributions to the shear strength of sandy soils.

Beside the intrinsic (φ_{μ}, or φ_{cv} and σ'_c) and state (D_R and σ'_{mf}) parameters, which mainly influence the shear strength of granular materials, other aspects can also play a significant role under some specific conditions.

Among them, the first to be mentioned is the intermediate principal stress σ'_2 (see Fig. 11) that has been already introduced in an indirect way referring to PS and TX strain conditions.

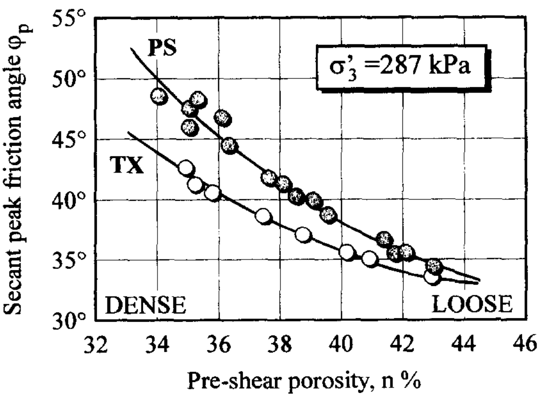

Figure 11. Comparison of φ_p (PS) with φ_p (TX) for a siliceous sand (Cornforth, 1964)

Most of the laboratory experimental data are the output of tests in the triaxial apparatus, whereas many geotechnical structures work in plane strain conditions. It can be, therefore, of practical interest to transform φ_p(TX) in φ_p(PS) or vice-versa, by means of a number of empirical formulae reported in the geotechnical literature. Lade and Lee (1976), for siliceous sands have suggested:

$$\varphi_p(PS) = 1.5 \cdot \varphi_p(TX) - 17° \qquad \varphi_p(TX) \geq 34° \tag{13}$$

Schanz & Vermeer (1996), considering the results on Hostun sand as well as those obtained by Cornforth (1964) and Leussink (1996), for siliceous sands have proposed:

$$\varphi_p(TX) = (1/5)(3\varphi_p(PS) + 2\varphi_{cv}) \tag{14}$$

This latter equation is exactly the same as that which can be obtained by equation (11) of Bolton (1986) evaluated for m=5 and m=3 in (PS) and (TX) conditions respectively.

Both Eqs. 13 and 14 hold when comparing TX and PS compression tests. A more general handling of the problem regarding the effects of σ'_2 on shear strength of granular soils must include the full range of variation of the following parameter: $b = (\sigma'_2 - \sigma'_3)/(\sigma'_1 - \sigma'_3)$. However, the analysis of the changes of φ_p with variation of b is beyond the scope of the present paper.

As documented in the last fifteen years by Tatsuoka and his co-workers (Tatsuoka et al., 1986, 1990; Pradhan et al., 1988; Park & Tatsuoka, 1994), the peak secant angle of shearing resistance shows a pronounced anisotropic response. As a matter of fact, in addition to the value of parameter, b, the peak friction angle is also affected by the angle, δ, existing between the direction of the major principal stress at failure (σ'_{1f}) and the direction of the bedding planes. The present φ_p anisotropy is not usually taken into account in the interpretation of in situ tests results for strength.

The previous approaches to the assessment of the shear strength envelope of coarse grained materials assume that there is no cohesion intercept (c') in terms of effective stress.

Such statement holds even in very dense and interlocked materials as recently argued by Schofield (1998).

However, lightly cemented coarse-grained soils are noticed within natural formations for which a c'>0 intercept is a consequence of the weak bond between the soil grains (Nader, 1983; Bachus, 1983).

The lightly cemented soil deposits generally have unconfined compressive strength less than 100 kPa and the c' resulting from drained TX compression tests falls in a range between 5 and 30 kPa.

At present there is a lack of well consolidated methods allowing to infer both φ'_p and c' from in situ tests, while some possibilities can be envisaged for SBPT (Bachus, 1983; Carter et al., 1986).

Currently the assumption of c'=0 when interpreting in situ CPT and DMT leads to an overestimate of φ'_p in case of lightly cemented sands.

Evaluation of the peak friction angle from CPT and DMT. Nowadays the CPT and DMT are among the tools most commonly used in design to evaluate φ_p. With this respect two basic different approaches can be envisaged (Jamiolkowski & Lo Presti, 2000):

A. The first approach can apply to both CPT and DMT results and refers to the use of the existing bearing capacity theories (e.g. Durgunoglu & Mitchell, 1975; Janbu & Senneset, 1974; Vesic, 1975, 1977; Salgado, 1993, Salgado et al., 1997). In this case, the ultimate bearing capacity is measured (i.e. the cone resistance q_c in case of CPT and the wedge resistance q_D in case of DMT), therefore, the bearing capacity formula is used to estimate φ_p. The summary of the input data required when using these approaches is shown in Table 9. Further details can be found in the works by Mitchell & Keaveny (1986) and Yu & Mitchell (1998).

B. The second approach consists of the in situ evaluation of D_R from the results of the considered penetration tests. Once the D_R has been assessed, the estimation of φ_p can be carried out by using correlations φ_p= f(D_R, grading) like the one proposed by Schmertmann (1978), see Fig. 12 or, in a more refined manner, by means of an iterative use of Bolton's (1986) stress dilatancy Eqs. (11) and/or (12).

The main features and the use of the methods belonging to the groups A and B are summarized in the paper of Jamiolkowski & Lo Presti (2000).

As a matter of fact, the methods for estimating φ_p by the bearing capacity theories, in spite of the more elegant initial approach, require rather complex input data and /or are affected by important limitations and approximations of the original theoretical models so that, most of the procedures, practically applicable, must turn to "calibrating" coefficients and/or "operational" parameters that reintroduce empiricism into the initial equations (see for example Mitchell & Keaveny, 1986).

For such reasons the methods of group B, passing from the empirical evaluation of D_R by CPT and DMT before the final assessment of φ_p at different confining stress levels, can still be considered more robust, reliable, and useful for practical applications within the current geotechnical design practice.

Referring to the φ_p assessment by the methods of the group B, once D_R has been evaluated by means of one of the approaches outlined in the first section of this paper, an estimate of the φ_p(TX) can be attempted with reference to Fig. 12 by selecting the line appropriate for the gradation curve for the soil layer in question. The main limitation which arises from the use of the functions of Fig. 12 is that φ_p only refers to triaxial and direct shear (DS) tests carried out at a confining stress range of $50<\sigma_{oct}<350$ kPa and at a normal stress range of $80<\sigma'_{vc}<400$ kPa respectively. Therefore, it is not possible to take into account the influence of parameters such as: different confining stress level at failure, strain conditions and last but not least sands characterized by different compressibility and mineralogy from the tested ones.

In order to overcome these limitations, Eqs. 11 or 12 can be used to assess φ_p(TX) or φ_p(PS) following the procedure outlined in Fig. 13. This approach has

Table 9. Friction angle from cone resistance – possible approaches

REFERENCE	STRESS-STRAIN RELATIONSHIP	COMPRESSIBILITY	STRENGTH ENVELOPE	STRESS TENSOR	OTHER INPUT REQUIRED
SCHMERTMANN (1978)	NO ASSUMPTION EMPIRICAL	NO	LINEAR	σ'_{v0} or σ'_{m0}	D_R GRADING CURVE
BEEN ET AL. (1987)	NO ASSUMPTION	YES (IMPLICITLY)	LINEAR	σ'_{m0}	$\xi = f(NCR)_0$
BOLTON (1986)	STRENGTH-DILATANCY THEORY SEMI-EMPIRICAL	YES (IMPLICITLY)	CURVILINEAR	σ'_{mf}	Q, φ'_{cv}
JANBU AND SENNESET (1974)	RIGID PLASTIC	EMPIRICALLY	LINEAR	σ'_{v0}	s_q
DURGUNOGLU AND MITCHELL (1975)	RIGID PLASTIC	NO	LINEAR	σ'_{v0}	s_q, K_0, δ
VESIC (1972, 1977)	ELASTIC PERFECTLY PLASTIC	YES	LINEAR	σ'_{m0} or σ'_{h0}	s_q, K_0, G
SALGADO (1993)	ELASTIC NON LINEAR PLASTIC	YES	CURVILINEAR	σ'_{h0}	$s_q, \varphi'_{cv}, K_0, D_R, Q, G(\gamma, \sigma'_m)$

The table is not exhaustive, for similar approaches see also Robertson & Campanella (1983), Jamiolkowski et al. (1988), Trofimenkov (1974)

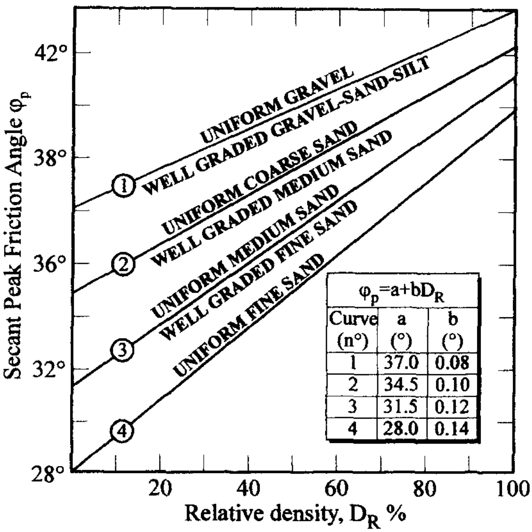

Figure 12. Correlation $\varphi_p = f(D_R$ Grading) (Schmertmann, 1978)

been validated for a number of siliceous sands by Bellotti et al. (1989), Jamiolkowski (1990), Yoon (1991) and others. This method is equally conditioned by the reliability of the D_R best estimate as that involving the use of Fig. 12, but has the advantage to be able to take into account the stress level relevant to the considered boundary value problem via the introduction of an appropriate value of σ'_{mf}. The higher rationality of this approach has, however, the limitation of a more elaborated input involving the knowledge and/or the assumption, in addition to D_R, of two intrinsic (φ_{cv} or φ_μ and Q or σ'_c) and one state (σ'_{mf}) parameter.

As far as $Q=\ln\sigma'_c$ is concerned, Table 8 and 10 report respectively the values of Q as suggested by Bolton (1986) and those resulting for a number of sands inferred from triaxial compression tests. Table 10 associates with each value of Q also the corresponding φ_{cv}. In the light of the preliminary considerations related to the possible difference of 2° existing between φ_{cv}(TX) and φ_{cv}(PS), in the authors opinion, the value reported in Table 10 should be considered as φ_{cv}(TX). Moreover it may be worthy to point out that the values of Q displayed apply to grains having dimensions corresponding to those of fine to medium sands. As documented by Lee (1992) for coarse sands and gravel particles constituted by the same mineral, Q tends to decrease with increasing the equivalent grains diameter.

Regarding the estimate of σ'_{mf} corresponding to the boundary value problem of practical interest, the issue is far from being solved in a rigorous manner. At present, only the following rules of thumb can be suggested to the readers:

For shallow foundations (De Beer, 1967):

$$\sigma'_{ff} \cong \frac{q_{lim} + 3\sigma'_{v0}}{4}(1 - \sin\varphi_{op}) \tag{15}$$

q_{lim} : limit bearing capacity of the considered shallow foundation;
φ'_{op} : operational friction angle for q_{lim} evaluation.

For deep foundations and penetration tests (Fleming et al., 1992):

$$\sigma'_{ff} \cong \sigma'_{v0} \cdot \sqrt{N_q} \tag{16}$$

N_q : dimensionless bearing capacity factor for deep foundation, function of φ'_{op}.

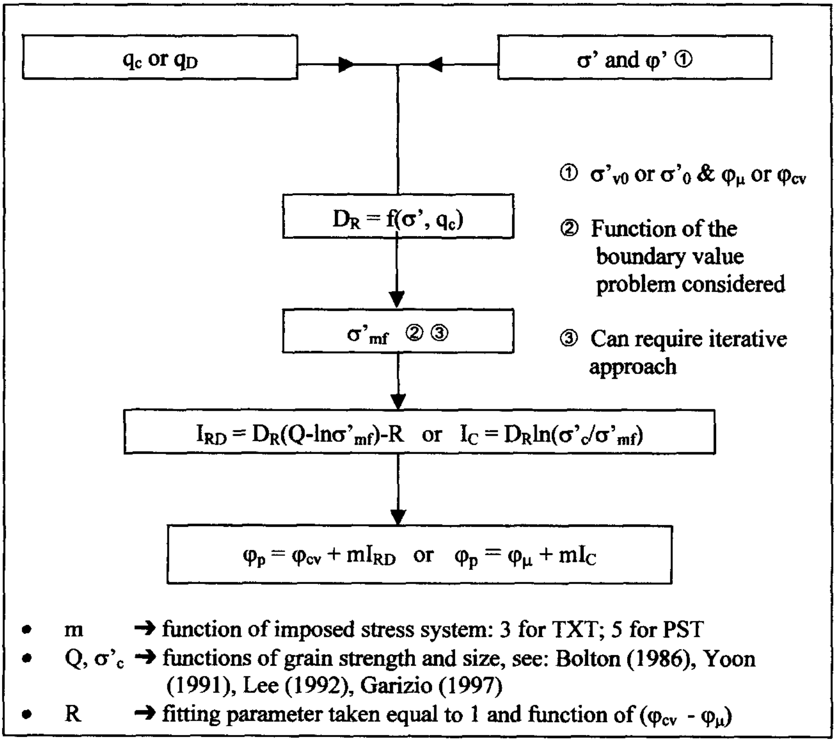

Figure 13. Friction angle of sand from penetration tests Bolton's (1986) stress-dilatancy theory

Table 10. Q-values of different uniform sands (*)

SAND	MINERALOGY		Q	φ_{cv}	REFERENCE
TICINO	SILICEOUS (**)		10.8	34.6	
TOYOURA	QUARTZ		9.8	32	Jamiolkowski et al., 1988
HOKKSUND	SILICEOUS		9.2	34	
MOL	QUARTZ		10	31.6	Yoon, 1991
OTTAWA	QUARTZ	FINES 0%	9.8	30	Salgado et al. 1997 Salgado et al. 2000
		FINES 5%	10.9	32.3	
		FINES 10%	10.8	32.9	
		FINES 15%	10	33.1	
		FINES 20%	9.9	33.5	
ANTWERPIAN	QUARTZ & GLAUCONITE		7.8 to 8.3	31.5	Yoon, 1991
KENYA	CALCAREOUS		9.5	40.2	Jamiolkowski et al., 1988
QUIOU	CALCAREOUS		7.5	41.7	

(*) inferred from TX compression tests
(**) i.e.: containing a comparable amount of quartz and feldspar grains

The link between σ'_{ff} and σ'_{mf} for compression loading stress path is given by:

$$\sigma'_{mf} \cong \sigma'_{ff} \cdot \left(\frac{3 - \sin\varphi_{0_p}}{3\cos^2 \varphi_{0_p}} \right) \tag{17}$$

As already pointed out, the method, based on the Bolton (1986) theory, sketched in Fig. 13 has been mainly validated referring to silica sands. An attempt has been carried out, by the writers, to extend the aforementioned procedure, for the assessment of D_R and then φ_p from q_c values, to sands characterized by different mineralogical compositions that results mainly in a different deformability under isotropic stress increments and in different values of φ_{cv} and φ_μ.

As a first step, the relationships q_c - D_R, reported in Fig. 4, has been considered. These correlations have been worked out by Lancellotta (1983) and subsequently have been revised by Garizio (1997) for taking into account the influence of CC dimensions, geometry and boundary conditions and referring to a much more extended data base.

To account for the influence of different sand compressibilities, the fitting parameters A_0 and B_0 of the equation in Fig. 4 have been evaluated for the average, upper and lower bond respectively of the data set displayed in the same figure.
This data set is mostly related to different normally-consolidated (NC) and lightly over-consolidated (OC) sands for which, as previously mentioned, D_R may be related to q_c through σ'_{vo}.

In order to be able to include, within the proposed procedure, the OC sands, as a first attempt, the equation of Figure 4 has been modified as reported in Table 11, i.e. instead of the geostatic vertical stress (σ'_{vo}) the relative density has been expressed as a function of the mean geostatic effective stress [$\sigma'_{mo}=\sigma'_{vo}(1+2K_0)/3$)] and the A_0 value has then been transformed by using the following equation:

$$A'_0 = A_0 - B_0 \ln\sqrt{\frac{1-2K_0(NC)}{3}} \tag{18}$$

where $K_0(NC)$ is the coefficient of horizontal pressure at rest for NC sands assumed, in a first approximation, equal to 0.4.

The proposed modification of the equation of Figure 4 into the one of Table 11 does not change the results of the original correlations in the case of NC sands but via the K_0 values allows, in principle, to extend the proposed procedure to the OC sands.

The obtained values of A'_0 and B_0 for the different mineralogical compositions are reported in Table 11 together with the values of Q characterizing the same kind of sands as suggested by Bolton (see also Table 8).

Substituting the equation reported in Table 11 into Eqs. 11 or 12 and using the parameters given in Table 11 and the values of φ_μ or φ_{cv} within the ranges of

Table 11. Parameters A'_0, B_0 and Q used in Figs. 14, 15 and 16 for different sands

	MINERALOGICAL COMPOSITION	COMPRESSIBILITY	EXAMPLES IN THE LITERATURE	A'_0	B_0	Q
QUARTZ SANDS	QUARTZ	LOW	MONTEREY OTTAWA TOYOURA SYDNEY	-1.506	0.268	10.0
SILICA SANDS	FELDSPAR QUARTZ MICA	MEDIUM	TICINO HOKKSUND	-1.360	0.268	9.5
CALCAREOUS SANDS	SANDSTONE MICA	HIGH	QUIOU KENYA BASS STRAIT ANTWERPIAN CHATTAHOOCHEE	-1.214	0.268	8.5

$$D_R = A'_0 + B_0 \ln\left(\frac{q_c}{\sqrt{\sigma'_{m0}}}\right); \quad q_c,\ \sigma'_{mo} \quad \text{in kPa}$$

Table 12. Range of φ_{cv} e φ_{μ} values for sands of different mineralogical compositions

	φ_{μ}	φ_{cv}
Quartz sands	25° ÷ 30°	30° ÷ 34°
Silica sands	27° ÷ 32°	32° ÷ 36°
Calcareous sands	32° ÷ 38°	36° ÷ 42°

Table 12, the peak friction angle φ_p(TX) or φ_p(PS) can be evaluated for the considered sand.

Figures 14, 15 and 16 show φ_p(TX) trends, for silica, calcareous and quartz sands respectively, evaluated by the proposed procedure and referred to a mean confining stress at failure σ'_{mf} equal to the geostatic mean effective stress σ'_{mo}. In the same figures the adopted values of φ_{cv} and Q are also reported.

The values of φ_p(TX) obtained from the aforementioned figures have been compared with the estimation of the same parameter at the same confining stress levels carried out in different ways by Durgunoglu & Mitchell (1975), Robertson & Campanella (1983) and Chen & Juang (1996), The comparison results have pointed out a very good agreement for all the mineralogical compositions of the considered sands.

Once the set of intrinsic and state parameters, characterising the mechanical behaviour of the considered sand, has been assessed, it is possible to evaluate the φ_{op}(TX) and/or the φ_{op}(PS) values for different confining stress levels at failure by using the Bolton (1986) Eqs. 11 or 12 and referring to Eqs. 15, 16 and 17 for some typical boundary value problems. However, since σ'_{mf} is a function of q_{lim} that in turn depends on φ_{op}, a trial and error procedure must be adopted.

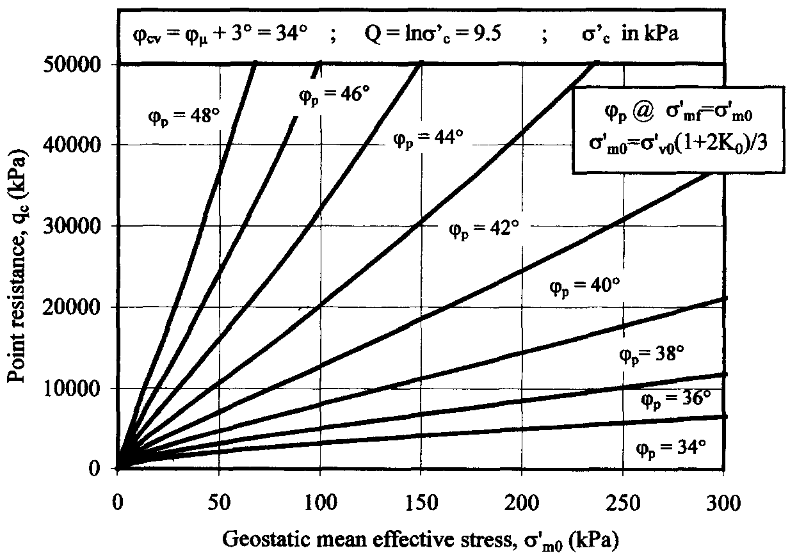

Figure 14. Peak friction angle from CPT for silica sands using Bolton (1986) theory

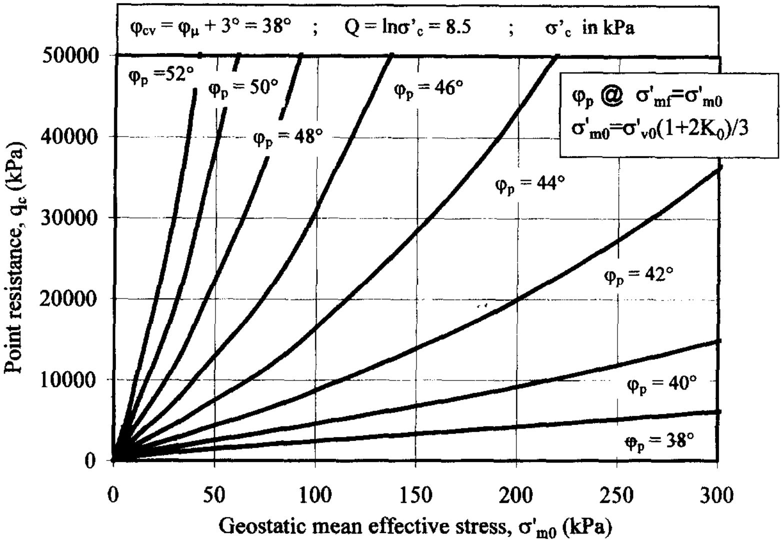

Figure 15. Peak friction angle from CPT for calcareous sands using Bolton (1986) theory

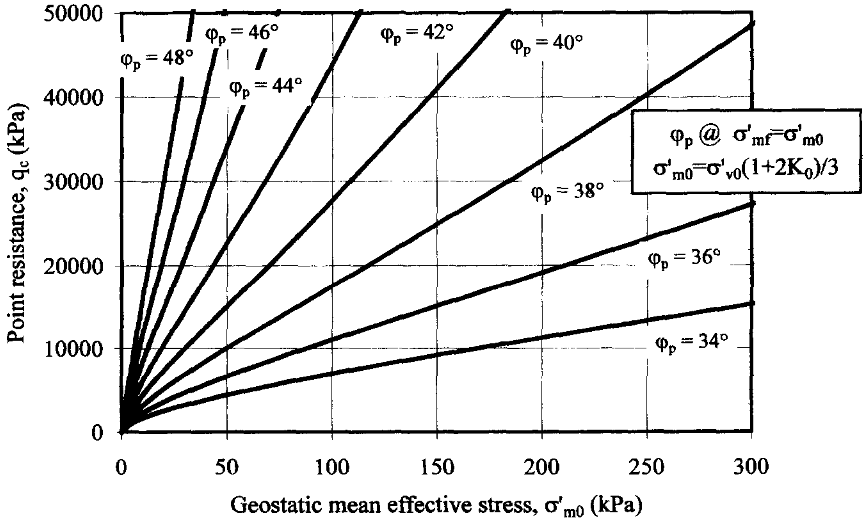

Figure 16. Peak friction angle from CPT for quartz sands using Bolton (1986) theory

For the estimation of q_{lim} and N_q of Eqs. 15 and 16 it is recommended to adopt $\varphi_{op}=(\varphi_p+\varphi_{cv})/2$ so as to take also into account, beyond the effect of the curvature of the strength envelope, the influence, on the mobilized average friction angle, of the strain level in correspondence of the limit pressure of the considered boundary value problem (i.e. progressive failure effect).

Finally, it is worthy to recall that the proposed procedure for the estimation of φ_p and φ_{op} allows also to consider the over-consolidation ratio (OCR) of the considered sand via the K_o value. Unfortunately, at the present state of the art, the quantification of the OCR degree and then of K_o, within coarse grained materials, is still very unreliable.

Closing remarks

1. Eq. 5, referring to the empirical coefficient C_0, C_1 and C_2, shown in Table 4, allows the estimation of D_R in deposits of unaged, uncemented silica sands of low to moderate compressibility.
2. The use of the above equations involves the effective overburden stress σ'_{vo} in case of NC sands but requires the estimation of the mean geostatic stress σ'_{mo} for OC deposits.
3. An alternative approach for the evaluation of D_R by Lancellotta (1983) and later reworked by Garizio (1997) is displayed in Fig. 4. This correlation, using the available data-base for both NC and OC sands makes reference to σ'_{vo}.
4. Making reference to the above mentioned $D_R = f(q_c, \sigma')$ correlation, an iterative approach based on the Bolton's strength-dilatancy theory is proposed to estimate φ'_p on the basis of the CPT results.
5. The proposed approach, at least in principle, allows evaluation, for a given sand, of φ'_p taking into account: sand compressibility, curvilinear nature of the shear strength envelope and imposed strain conditions.
6. Figures from 14 to 16 display the correlations $\varphi'_p = (q_c,\sigma'_{mo})$ for three qualitatively defined classes of sand compressibilities assuming $\sigma'_{mf} = \sigma'_{mo}$. For σ'_{mf} different than the σ'_{mo} iterative procedure outlined in Fig. 13 should be followed.
7. A comparison between $\varphi'_p = f(q_c,\sigma')$ yielded by Fig. 14 and φ'_p, obtained from drained compression loading CK_oD triaxial tests, suggests that the proposed procedure tends to underestimate φ'_p by 1° to 1°½, see Jamiolkowski (1990).
8. On the basis of the aforementioned approaches, an iterative procedure has been suggested for the estimation of the operational friction angle φ'_{op} referred to some current limit equilibrium boundary value problem. This approach allows also to take into account, in a first approximation, the progressive failure aspect.
9. This paper gives also engineering correlations between Marchetti's (1980) dilatometer lateral stress index K_D, D_R and φ'_p respectively as inferred from CC tests on siliceous sands.

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