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Effects of Stress History on Deformation of Sand

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Introduction

Prediction of the behavior of granular soils under applied loadings is a common problem facing the geotechnical engineer. Most frequently it is the deformation response of the sand, not its stability, that is of greatest concern. Of the many factors that influence a granular material's stress-deformation behavior, past loading history is the most significant in terms of its potential for altering performance. It is, however, not easy to ascertain the nature of such alterations using presently known methods.

The influence of various physical characteristics of sand on its compressibility has been investigated by many researchers who have shown that: (1) At the same relative density a well-graded sand is more compressible than a uniformly graded one (42,47); (2) compressibility decreases slightly with increasing grain size (2); (3) as particle angularity increases compressibility also increases both during initial loading (19,23) and repeated loading (10,20); (4) an assemblage of rough particles is less compressible than one of smooth particles (40,49); and (5) that a sand's deformation response is directly related to the parent mineral's compressibility (19,39). Extreme differences in certain of these physical properties between two sands may yield a fourfold difference in compressibility.

While the influence of each physical factor on load-deformation behavior has been quantified, the in-situ fabric of cohesionless soil is not readily assessible. Moreover, the manner of bedding, which often is not uniform or continuous, also strongly influences behavior. Obliquities between bedding and loading planes can cause more than a twofold difference in compressibility (3,26,42,43). Large differences in the orientation and thickness of layers and in the physical

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characteristics of the granular material itself exist over fractions of inches (4,36,44) and pose enormous difficulties in assessing the performance of naturally occurring granular deposits. In recent years engineers have turned increasingly to continuous cone penetration records as a primary tool to assist with the interpretation of the effects of these variables on in situ behavior (14,38,46).

Density has long been used as an averaging parameter from which the behavior of sand under applied loading has been inferred. The compressibility of cohesionless soil has been shown to increase with decreasing density for *all* of the methods of stress application that have been examined (1,9,11,17,18,27,29,33). Between readily obtainable limiting void ratios, a sand's compressibility may vary by a factor as great as five (2,12,13,16).

Although density may significantly influence the initial compressibility of a granular soil its importance is overshadowed by previous loading history. Using

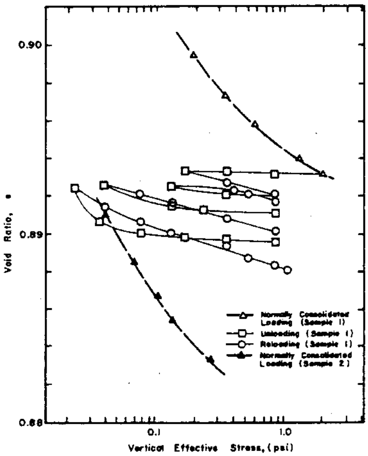

FIG. 1.—Relative Influence of Density and Stress History on Compressibility of Sand (after Yoshimi, Kuwabara and Tokimatsu, Ref. 51) (1 psi = 6.9 kN/m^2)

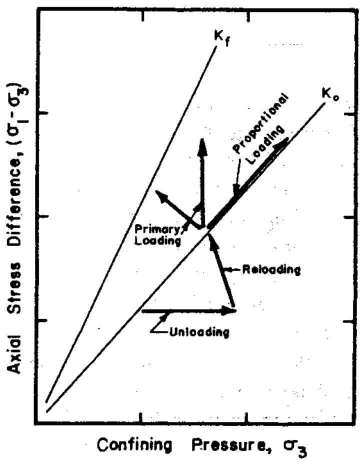

FIG. 2.—Stress Path Criterion for Mode of Deformation (after Lade and Duncan, Ref. 35) (1 psi = 6.9 kN/m^2)

an ingenious adaptation of the quicksand tank, Yoshimi, Kuwabara, and Tokimatsu (51) reproduced uniform void ratios accurately in large samples using a controlled upward flow of water. By reversing the direction of water flow, they could induce one-dimensional loading over the entire sample. The results for two samples are shown in Fig. 1. After initial loading, sample 1 was cyclicly unloaded and reloaded, there being a large difference in compressibility between the initial loading and reloading behavior. A second sample was prepared to a void ratio nearly equal to that of the prestressed sample. Upon initial loading the normally consolidated sand sample was at least six times more compressible than the prestressed sand even though their initial densities were essentially equal. No other parameter or material characteristic (except extreme ranges in relative density) has been shown to have such a profound effect on the compressibility of granular soils.

The differentiation between initial loading and reloading compressibility is generally linked to the relative magnitudes of the elastic and plastic deformations that occur (5,22,28,37). Until recently, no method was available for predicting at what state of stress reloading would end and virgin loading deformation behavior would resume when more than one principal stress was varied. Lade and Duncan (34,35) have proposed an elastoplastic theory for modeling the stress-strain behavior of sand that attempts to account for the effects of prestressing resulting from changes in any of the three principal stresses. As shown in Fig. 2, the stress ratio, $K = \sigma_1/\sigma_3$ is used as a basis in formulating a criterion for the mode of a deformation. Proportional loading, modeled as a totally elastic process, occurs when the stresses change such that the stress ratio remains constant. Unloading, also modeled as being totally elastic, is experienced whenever the stress ratio decreases, even if there is an overall stress increase. Reloading is modeled as being totally elastic and is said to occur whenever K increases but remains less than the past maximum value experienced by the sand. Primary or virgin loading is experienced only when the stresses change such that K exceeds its past maximum value. The criterion proposed by Lade and Duncan was a major step forward; it is the only useful basis for defining virgin loading, unloading, and reloading modes now extant.

Lade and Duncan's (35) comparison of their predicted and actual stress-strain behavior shows generally close agreement for those stress paths investigated. Deviations from the predicted response are evident in some cases where the reloading stress path differed from the unloading stress path. The present investigation examines the factors that determine whether a sand will deform under virgin loading or reloading conditions, and whether the deformation mode—and, thus, the modulus—can be sensed with a cone penetrometer.

Apparatus and Procedures

Cylindrical triaxial specimens, on which the intermediate and minor principal stresses were considered equal, were used in this study. The samples averaged 6.20 in. (157.5 mm) in length and 2.80 in. (71.1 mm) in diameter. Sample preparation generally followed the procedures outlined by Bishop and Henkel (8). The "raining method," in which dry sand was allowed to fall from a funnel and form a conical pile in the center of the mold, was used in placing the dry sand. Because the density obtained by the raining process is sensitive to minor variations in the height of fall and the mass flow rate (50), these factors were strictly controlled at 3.5 in. (89.9 mm) and 80 g/min, respectively.

The sand used was a uniformly graded Ottawa quartz sand [American Society for Testing and Materials (ASTM) C109-59] all of which passed the No. 40 sieve with 80% being retained on the No. 50 sieve and the remainder being retained on the No. 60 sieve. The grains were well rounded and equidimensional with "frosted" surfaces. The limiting void ratios were found to be $e_{min} = 0.545$ and $e_{max} = 0.868$. Sample void ratios were all within ±0.005 of 0.685, which corresponds to an average relative density of 57%.

Stresses were applied to the samples under a controlled stress regime. Controlled stress tests were selected because the compressibility of sand is strain rate dependent and it was desired to measure quasi-static effects. Sample confinement was effected by compressed air acting on samples contained within rubber membranes 0.005 in. (0.127 mm) in thickness. Most of the samples were tested using sintered brass porous stones for end platens. The effects of roughened end platens were evaluated by comparison with tests using lubricated end platens (45).

To monitor lateral deformation, a two-armed feeler system incorporating an Linear Variable Differential Transformer (LVDT) was used. The sensitivity of the LVDT was greater than 10^{-6} in. (25.4×10^{-6} mm), so that the initiation of lateral deformation could be monitored accurately. However, inherent errors

TABLE 1.—Explanation of Stress Path Segments

	STRESS CONDITIONS				
	Beginning of Segment		End of Segment		
Stress path segment (1)	σ_3, in pounds per square inch (2)	$\sigma_1 - \sigma_3$, in pounds per square inch (3)	σ_3, in pounds per square inch (4)	$\sigma_1 - \sigma_3$, in pounds per square inch (5)	Stress changes during stress path (6)
$\overline{AC}$	2.0	2.3	5.3	6.1	K_0, stresses proportionally increased
$\overline{BB'}$	8.5	9.8	24.0	27.7	K_0, stresses proportionally increased
$\overline{B'B''}$	24.0	27.7	60.0	69.2	K_0, stresses proportionally increased
$\overline{CB}$	5.3	6.1	8.5	9.8	K_0, stresses proportionally increased
$\overline{CD}$	5.3	6.1	8.5	6.1	$\sigma_1 - \sigma_3$ held constant, σ_3 increased
$\overline{BC}$	8.5	9.8	5.3	6.1	$K = 2.15$, stresses proportionally decreased
$\overline{BD}$	8.5	9.8	8.5	6.1	σ_3 held constant, $\sigma_1 - \sigma_3$ decreased
$\overline{DE}$	8.5	6.1	8.5	3.1	σ_3 held constant, $\sigma_1 - \sigma_3$ decreased
$\overline{EC}$	8.5	3.1	5.3	6.1	σ_3 decreased, $\sigma_1 - \sigma_3$ increased
EF	8.5	3.1	8.5	2.7	σ_3 held constant, $\sigma_1 - \sigma_3$ decreased

Note: 1 psi = 6.9 kN/m^2

due to irregular sample shape and membrane effects precluded precise determination of lateral strain.

During initial loading of normally consolidated sand samples there is only one stress path along which, under increasing stresses, the conditions of no lateral strain may be maintained, i.e., the K_0 line. However, many different stress paths exist for which no lateral sample deformation will occur when the sample is unloaded and reloaded (52). As used herein, K_0 stress conditions are those states of stress corresponding to initial loading only. Upon unloading and reloading, reference may be made to a stress ratio equivalent to K_0 in virgin loading irrespective of whether or not lateral strain is occurring. Tests showed that stress ratio corresponding to K_0 conditions was $\sigma_1/\sigma_3 = 2.15$. The stress ratio at no lateral strain is the reciprocal of the coefficient of earth pressure at rest. The constraints of sample preparation made it necessary initially to confine the samples under an isotropic stress of 2.0 psi (13.8 kN/m^2) and then increase the axial stress to bring the samples to the K_0 stress conditions.

Small stress increments were used during non-K_0 loading to enable better definition of the stress-strain response of the samples; axial stress increments of less than 10% of the confining pressure were employed. Because sands do

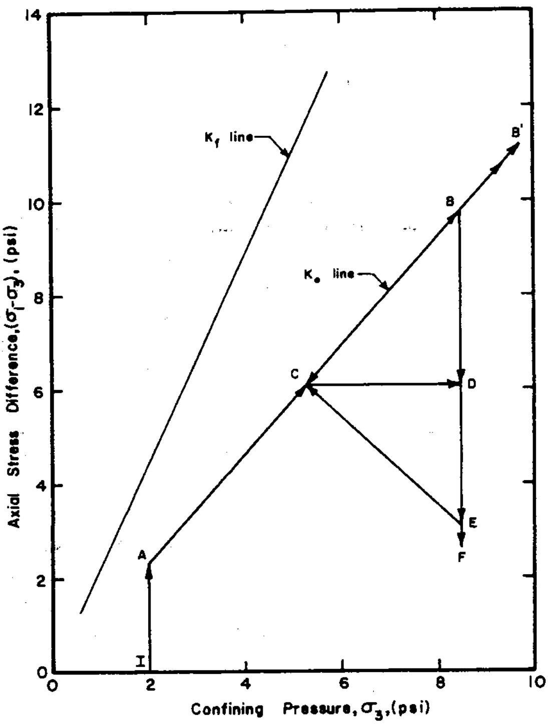

FIG. 3.—Stress Path Segments (1 psi = 6.9 kN/m^2)

exhibit significant short-term creep characteristics, each increment of axial load was allowed to remain for 5 min before the axial strain reading was made or the next increment of load applied.

Testing Program

In determining the effect of stress history on the stress-deformation characteristics of the sand under study, stress paths simulating a variety of stress histories were employed. At the end of each such stress path, the axial stress was increased while maintaining a constant confining pressure, and the initial and subsequent slopes of the stress-strain curves were determined. These slopes were defined as moduli of deformation of the sand. By comparing the modulus values and the ranges in axial stress difference over which they apply, in reference to the respective stress history, the influence of stress path on the stress-deformation behavior of the samples was evaluated.

Each of the stress paths used in simulating different stress histories was composed of combinations of stress path segments that are shown in Fig. 3 and explained in Table 1. Stress paths are generally designated by a sequence of letters that represent the stress conditions at the ends of the stress path segments which jointly comprise a sample's stress history. Because all of the samples were similarly stressed along stress path segment $\overline{IA}$, it is not included in the stress path designations. The stress path segments $\overline{AC}$, $\overline{CB}$, and $\overline{BB'}$, are all at K_0 stress conditions. Therefore, a sample stressed along stress path ACB is initially normally consolidated.

Sample Replication

Sample preparation techniques were meticulously controlled to produce replicate samples. The initial void ratios were mostly within ±0.003 of 0.685. With such a close tolerance, initial void ratio variation was felt to have little effect, if any, on the stress-deformation characteristics of the samples tested. However, differences did exist between seemingly replicate samples as may be observed by examining the axial strains that occurred during initial sample stressing as tabulated in Table 2. Also tabulated are the initial void ratios of each sample. Positive strains indicate sample compression, and negative values refer to extension. No lateral deformation was permitted during initial stressing along stress path segments $\overline{AB}$ or $\overline{CB}$, which are along the K_0 stress path. Lateral strains were observed to occur during loading and unloading along other stress path segments.

From the axial strains presented in Table 2, it may be seen that differences in what otherwise might be thought to be replicate samples do exist. This demonstrates that minor differences in particle packing can affect the stress-strain behavior of uniformly graded sand samples, even when the void ratios have been essentially duplicated. Similar effects of minor differences in packing have been reported previously (31,42).

Behavior of Normally Consolidated Samples

Plots of axial stress difference, $(\sigma_1 - \sigma_3)$, versus axial strain for several of the normally consolidated samples are shown in Fig. 4. Insets on the figures are diagrams showing the stressing sequence undergone by each sample prior to initial axial loading. On the stress-strain plots, straight lines have been superimposed to represent equivalent linear approximations to the data; the slopes of these straight lines are called equivalent moduli. The first and second slopes are designated E_A and E_B, respectively, as shown in Fig. 4.

Practically all of the stress-strain plots of incremental axial loading, $\Delta\sigma_a$, exhibited quasi bilinear trends from which two distinct moduli could be obtained (as may be seen in Fig. 4). This behavior is due, in part, to the stiffening and restraining effect of the rough end platens. While not significantly influencing

TABLE 2.—Axial Strains Occurring Along Stress Path Segments

Test[a] designation (1)	Axial Strains, ϵ_a, as a percentage e_0 (2)	$\overline{AC}$[b] (3)	$\overline{CB}$[b] (4)	$\overline{CD}$ (5)	$\overline{BC}$ (6)	$\overline{BD}$ (7)	$\overline{DE}$ (8)	$\overline{EC}$ (9)	$\overline{EF}$ (10)
AC-1	0.683	0.157	—	—	—	—	—	—	—
AC-2	0.680	0.155	—	—	—	—	—	—	—
ACB-1	0.681	0.163	0.138	—	—	—	—	—	—
ACB-M-1	0.680	0.149	0.119	—	—	—	—	—	—
ACB-M-2	0.680	0.161	0.121	—	—	—	—	—	—
ACB-ML-1[c]	0.683	0.161	0.122	—	—	—	—	—	—
ACBC-2	0.690	0.188	0.132	—	−0.017	—	—	—	—
ACBC-3	0.684	0.155	0.143	—	−0.015	—	—	—	—
ACBC-4	0.680	0.183	0.142	—	−0.016	—	—	—	—
ACBDEC-1	0.688	0.163	0.131	—	—	−0.014	−0.018	0.009	—
ACBD-1	0.683	0.151	0.123	—	—	−0.014	—	—	—
ACBD-2	0.680	0.150	0.137	—	—	−0.012	—	—	—
ACBDEF-1	0.681	0.106	0.092	—	—	−0.006	−0.012	—	−0.002
ACD-1	0.687	0.149	—	0.014	—	—	—	—	—
ACD-2	0.690	0.188	—	0.016	—	—	—	—	—
ACD-3	0.680	0.148	—	0.014	—	—	—	—	—

[a] Refer to Fig. 4 for stress path designations.
[b] Stressing at K_0 conditions.
[c] Used lubricated end platens.

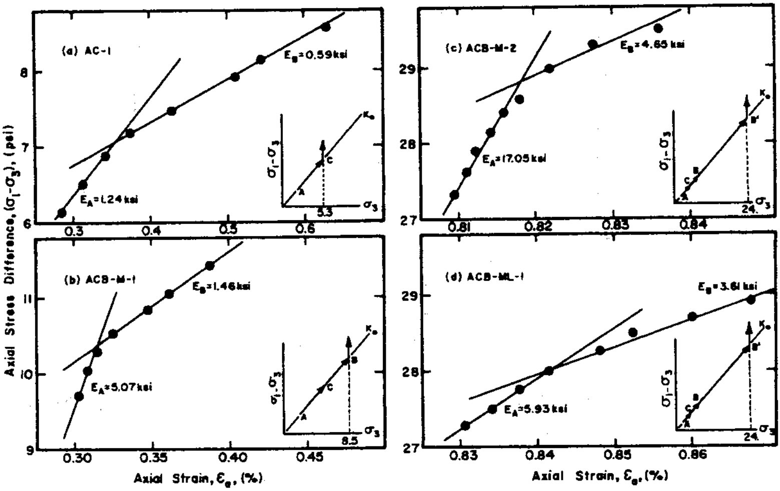

FIG. 4.—Strain Response of Axially Stressed Normally Consolidated Samples (1 psi = 6.9 kN/m²)

the shear strength of samples having length to diameter ratios greater than two to one, the restraint offered by nonlubricated end platens give the samples greater initial stiffnesses (7,32,45). The use of lubricated end platens greatly reduced this effect [compare Figs. 4(*c*) and 4(*d*)]. The bilinearity which is

TABLE 3.—Equivalent Moduli for Normally Consolidated Samples

Test designations[a] (1)	Confining pressure, σ_3, in pounds per square inch (2)	Initial equivalent modulus, E_A, in kips per square inch (3)	Second equivalent modulus, E_B, in kips per square inch (4)	$\Delta(\sigma_1 - \sigma_3)$ at yield point, in pounds per square inch (5)
AC-1	5.3	1.24	0.59	1.20
AC-2	5.3	1.13	0.69	1.65
ACB-1	8.5	6.20	1.33	0.775
ACB-M-1	8.5	5.07	1.46	0.60
ACB-M-1	24.0	17.24	4.94	1.20
ACB-M-1	60.0	11.96	11.96	—[b]
ACB-M-2	24.0	17.05	4.65	1.50
ACB-M-2	60.0	12.66	12.66	—[b]
ACB-ML-1	8.5	1.92	1.39	0.90
ACB-ML-1	24.0	5.93	3.61	0.70

[a] Refer to Fig. 4 for stress path designations.
[b] No yield point existed because there was only one slope to data.
Note: 1 psi = 6.9 kN/m^2; 1 ksi = 690 kN/m^2.

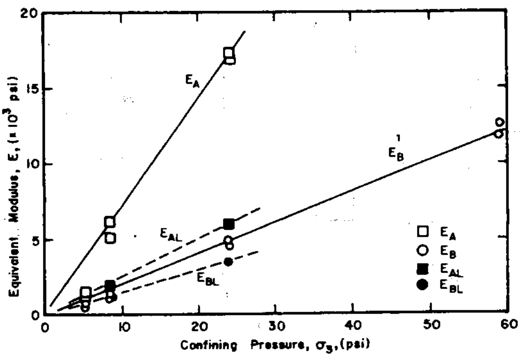

FIG. 5.—Equivalent Modulus Versus Confining Pressure for Normally Consolidated Samples (1 psi = 6.9 kN/m^2)

present with sample ACB-ML-1 is due to the inherent nonlinear nature of the stress-strain response of sand samples.

Presented in Table 3 are the values of the initial and second moduli, E_A and E_B, the corresponding confining pressure, σ_3, and the value of the increase in axial stress difference, $\Delta(\sigma_1 - \sigma_3)$, at which the modulus lines intersect (termed the "yield point") for each stress level at which a normally consolidated sample was tested. Shown in Fig. 5 is an arithmetic plot of the values of E_A and E_B versus σ_3; E_{AL} and E_{BL} refer to the moduli of the sample using lubricated end platens.

In Fig. 5, the E_{AL} and E_{BL} lines closely bracket the E_B line. Thus, for normally consolidated samples, the E_B line was taken to be the average initial modulus not influenced by end platen restraint. The following equation is the relationship between modulus and confining pressure as defined by the E_B line:

$$E_B = 200\,(\sigma_3) \qquad (1)$$

The equation most often used to relate modulus and confining pressure takes the following form (16,25,32, Lee, K. L., unpublished report):

$$E = m\,(\sigma_*)\left(\frac{\sigma}{\sigma_*}\right)^{\alpha} \qquad (2)$$

in which σ_* = reference stress, generally taken as atmospheric pressure; σ = measure of the state of stress; E = initial (constrained) modulus; and m and α = experimentally determined constants. For the stress conditions depicted in Fig. 5, $\alpha = 1.0$.

Prestressed Samples—Effect of K_0 Prestressing

A comparison of the axial stress difference versus axial strain curves for a normally consolidated sample and a proportionally loaded prestressed sample, both tested at $\sigma_3 = 5.3$ psi (36.57 kN/m^2), reveals that the prestraining associated with the K_0 prestressing increases the initial modulus by more than an order of magnitude (see Fig. 6). The stress-strain data presented in Fig. 6 are typical of the many samples tested that had been subjected to stress paths AC (normally consolidated) and ACBC (prestressed along the K_0 stress path). The criteria proposed by Lade and Duncan predict that both groups of samples would deform identically because the stress ratios never exceeded that value that existed immediately prior to the application of the first increment of $\Delta\sigma_a$.

The deformations that occur during K_0 stressing are largely irreversible (see Fig. 7 in which the K_0 loading of sample ACB-M-1 is shown), thus, proportional loading—like primary loading—is largely inelastic. This sample served as a "multistage" test, the normally consolidated modulus being measured at three levels of confining pressure. Because after each incremental $\Delta\sigma_a$ loading the sample's stress-strain behavior returned to the extrapolated K_0 virgin loading curve far in advance of the next stress level at which the equivalent modulus was determined, the sample was considered to be essentially unaffected by the previous $\Delta\sigma_a$ loading and, therefore, normally consolidated at the higher state of stress.

The permanence of the strain induced by K_0 loading, even for small prestress levels, is also evidenced by the data in Table 2. The compressive strains CB are nearly an order of magnitude larger than the rebound strains which accompany unloading along BC; however, the unloading strains are primarily reversible (elastic). There then exist many reloading stress paths along which the potential for permanent deformation has for the most part been removed by the prestressing (and associated prestraining) such that below some threshold stress, virtually no further plastic strain will occur. This is in agreement with the data recently presented by Ladd (30).

The state of stress at which reloading strain behavior ceases to be elastic is markedly influenced by the magnitude of the prestress. Shown in Fig. 8 are the stress-strain data of a normally consolidated sample, a moderately prestressed sample [prestressed at K_0 to σ_3 = 24 psi (165.6 kN/m^2)], and a highly prestressed sample [prestressed at K_0 to σ_3 = 60 psi (414 kN/m^2)],

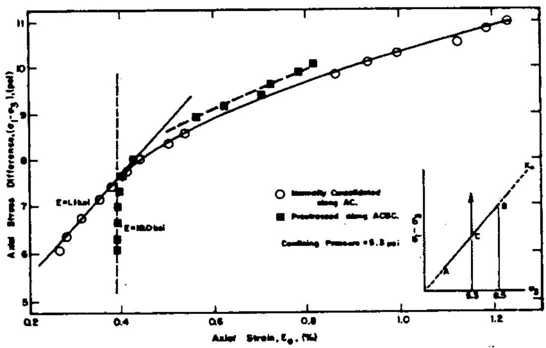

FIG. 6.—Effect of K_0 Prestressing on Deformation Response, Samples AC-2 and ACBS-4 (1 psi = 6.9 kN/m^2)

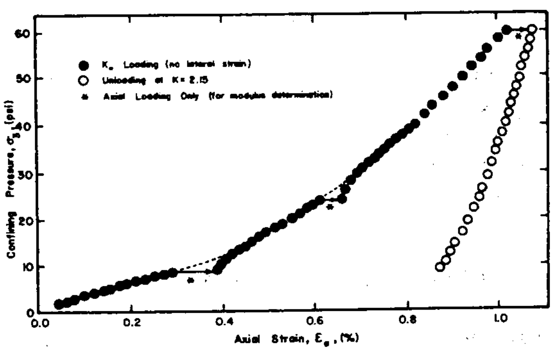

FIG. 7.—Permanent Strain Due to K_0 Prestressing of Sample ACB-M-1 (1 psi = 6.9 kN/m^2)

all at σ_3 = 8.5 psi (58.65 kN/m^2). As may be seen, the threshold stress is related to the magnitude of the prestress. In predicting the deformation behavior of prestressed sand, not only must the increase in modulus due to prestraining be forecast, but so must the threshold stress at which reloading deformation ends.

A graph of magnitude of prestress versus threshold stress, $\Delta(\sigma_1 - \sigma_3)$, that is here taken as the value of the axial stress difference *above* K_0 at which reloading deformation behavior ends, is presented in Fig. 9. As a measure of the magnitude of the prestress, an overconsolidation ratio (ORC) equal to the maximum σ_3 divided by σ_3 during axial loading has been used. Because during

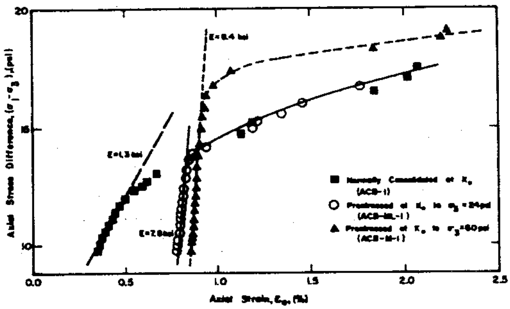

FIG. 8.—Effect of Prestress Level on Subsequent Load-Deformation Behavior, σ_3 = 8.5 psi (1 psi = 6.9 kN/m^2)

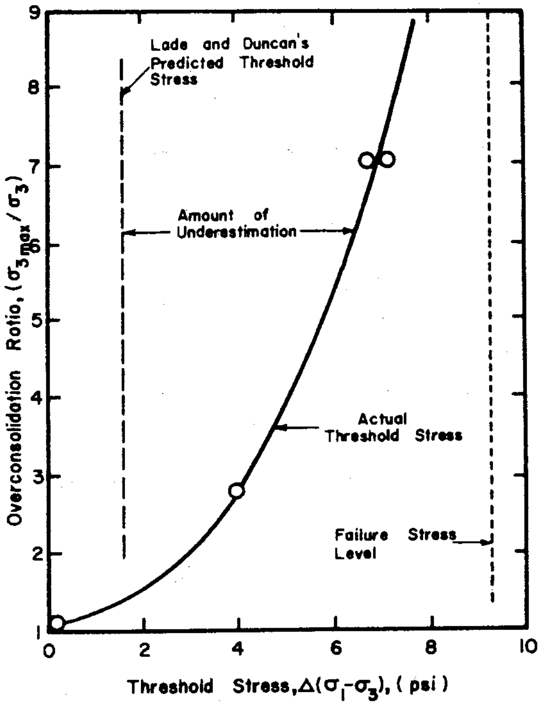

FIG. 9.—Threshold Stress as a Function of Overconsolidation Ratio, σ_3 = 8.5 psi (1 psi = 6.9 kN/m^2)

initial K_0 loading the samples were subjected to σ_a = 1.6 psi (11.0 kN/m^2) at σ_3 = 8.5, Lade and Duncan's criterion predicts a threshold (or yield) stress of equal value, as shown in the figure. As may be seen, the criterion established by Lade and Duncan for determining when virgin straining behavior recommences underestimates the threshold stress significantly at higher values of OCR [at OCR approaching 10, the stress ratio (σ_1/σ_3) approaches failure before primary loading recommences].

Prestressed Samples—Effect of Stress Path

The particular stress path followed during loading and unloading also has a significant influence on subsequent deformations. The effect of stress path during unloading is seen in Fig. 10 by comparing Figs. 10(*b*) with 10(*c*) in relation to 10(*a*). The unloading stress path BDEC effectively halves the modulus values obtained for path CBC. Thus, failure to account for the effect of stress

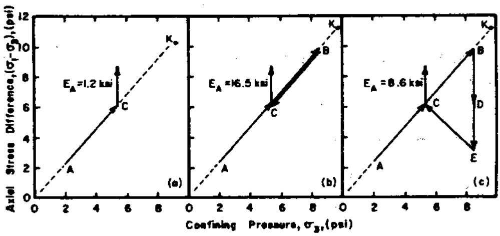

FIG. 10.—Effect of Prestress and Stress Path on Modulus (1 psi = 6.9 kN/m^2)

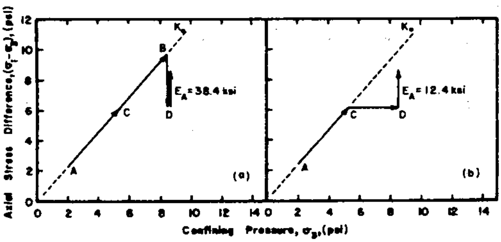

FIG. 11.—Effect of Stress Path and Residual Lateral Stress on Modulus (1 psi = 6.9 kN/m^2)

path in the unloading process could result in the reload modulus being in error on the order of 100%.

The effect that a residual lateral stress resulting from prestressing has on the initial modulus of prestressed samples may be seen by comparing Figs. 10(*a*) and 10(*b*) and Fig. 11(*a*). Prestressing alone increases the initial modulus by a factor of 14, while prestressing leaving a full residual lateral pressure further doubles the modulus. Additional evidence of the influence of stress path on compressibility may be seen in Figures 11(*a*) and 11(*b*). Stress path CBD results in an initial modulus three times larger than that from stress path CD alone. Thus, prestressing from C to D is not nearly as effective in increasing the subsequent initial modulus as prestressing from C to B to D, although the final state of stress is the same. It is evident that developing generally applicable constitutive relations for sands will prove to be a more difficult task than previously expected.

Shear Strength

Because all samples were failed under controlled stress conditions, the stress-strain curves became essentially flat at axial strains exceeding about 4%. The calculated angle of shearing resistance, Φ', of the sand from *all* of the samples tested was 32° ± 0.5° thus showing that previous stress history had no discernable effect on the shear strength (6,21,35). Using Jaky's (24) relationship

$$K_0 = 1 - \sin \Phi' \quad \text{. . . . . (3)}$$

the calculated range of K_0 is 0.478–0.463, the inverse of which agrees well with the experimentally determined value of 2.15.

In-Situ Evaluation of Deformation Modulus

Evaluation of the changes in stress-strain response due to prestressing was attempted using a model cone penetrometer. The triaxial apparatus was modified

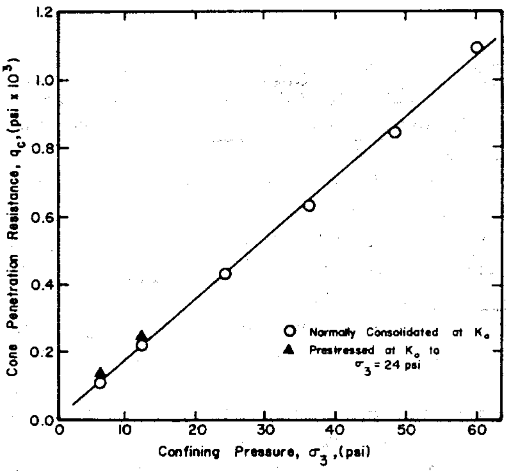

FIG. 12.—Cone Penetration Resistance Versus Confining Pressure (1 psi = 6.9 kN/m²)

to facilitate penetration of the sample through the base in a manner similar to that used by Thomas (48). The cone had a 60° apex angle and a base diameter of 0.5 in. (12.7 mm). A rate of penetration of 0.008 in./min (0.20 mm/min) was used to facilitate precise measurement of probe displacements; within wide limits, the rate of penetration has no influence on cone penetration resistance in dry sands (15).

The linear relationship between cone penetration resistance, q_c, and confining pressure is shown in Fig. 12. Penetrations were performed under K_0 stress conditions at several stress levels, including two states of K_0 prestress. As may be seen, K_0 prestressing increased the cone penetration resistance by less than 20%, although such prestressing increased the modulus of the sample by more than an order of magnitude.

It is the density and existing state of stress that largely control the magnitude of q_c. Recent field and laboratory data support the fact that the increase in q_c after prestressing of a sand is due largely to the residual lateral stress (14). Prestraining, without residual lateral stresses, has only a minor effect on cone penetration resistance but a very large effect on the deformation modulus.

Using Eq. 1 and the line relating q_c and σ_3 in Fig. 12, the equivalent modulus may be related to cone penetration resistance by:

$$E_B \simeq 11\, q_c \quad \text{(4)}$$

The most frequently reported range in the constant relating E to q_c for normally consolidated sands is 4.5–1.5 (38,46,48). Inherent test artifacts associated with the use of a small cone penetrometer, the sample's boundary stress conditions, and differences in the methods for evaluating E_B account for the reported discrepancies.

Summary and Conclusions

1. Minor differences in particle packing can affect the stress-strain behavior of a uniform sand, even when the void ratio has been precisely duplicated.
2. The initial modulus of normally consolidated sand was found to be essentially proportional to the confining pressure.
3. The strains that result from proportional (K_0) loading, like those during virgin loading, are largely inelastic.
4. The irreversible strain due to prestressing, including stress paths along the K_0 line, can increase the reload modulus by more than an order of magnitude over that of the normally consolidated sample, even though no residual stresses due to prestressing are extant. The effect of a residual lateral stress equal to that under K_0 preloading is to increase the modulus by an additional factor of about two.
5. The magnitude of the prestress that is applied and removed by K_0 stressing, and the associated prestraining, significantly influence the stress level at which the deformation behavior in the reloading mode ends and virgin loading begins; the higher the OCR, the higher the threshold (or yield) stress. Unloading along different stress paths to the same stress level does not result in the same subsequent reloading response. Thus, a stress ratio criterion defining the mode of deformation to be expected (virgin, unloading, or reloading) is not generally applicable.
6. Neither prior stress history nor minor differences in particle packing, which may influence stress-strain behavior very significantly, had any discernable effect on the shear strength of the sand samples tested.
7. The results of model cone penetration tests on triaxial sand samples having various stress histories show that the cone penetration resistance depends solely on the existing state of stress, the state of compaction, and the sand type involved. Thus, existing correlations between cone penetration and deformation modulus are applicable only to normally consolidated sands.

Appendix I.—References

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