Cone Resistance of a Dry Medium Sand

Résistance au Pénétromètre d’un Sable Moyen Sec


SUMMARY.

The paper presents a comparison between static cone resistance $q_c$ of dry dense and very dense medium sand measured during tests in a large calibration chamber with those computed on the basis of the theories proposed by Durgunoglu and Mitchell (1975) and by Vesic (1975, 1977). In the evaluation of cone resistance from the above mentioned approaches, the stress-strain-strength properties of sand determined in triaxial laboratory tests were used.

1. INTRODUCTION.

The paper presents some results of the research undertaken by ENEL-CRIS (Milano) and POLITECNICO di Torino with the aim to calibrate, under very carefully controlled conditions, the Electrical Fugro-type static penetration tip in sand.

![Diagram of calibration chamber](image)

**FIG. 1** - Scheme of the calibration chamber.

For this purpose a large calibration chamber has been developed: it houses samples 1.2 m wide and 1.5 m high and allows the performance of cone penetration tests (CPT) under selected boundary conditions.

A scheme of the calibration chamber and the boundary conditions are shown in Fig. 1 and 2 respectively. A detailed description of the apparatus used and of the stages of the test are given in Bellotti et al. (1979-a, 1979-b).

Alongside with the calibration of CPT tip, triaxial tests (TX) were carried out on the same sand used in the calibration chamber and prepared in the same manner, in order to determine its stress-strain-strength characteristics.

In this way it was possible to make a comparison, for N.C. sand, between cone resistance measured in the sand.
calibration chamber and that evaluated by means of some theoretical approaches [Vesic (1975, 1977), Durgunoglu and Mitchell (1972, 1975)], in which strength and deformability properties, determined by triaxial tests, were introduced.

2. SAND CHARACTERISTICS.

The sand used in the tests is described in fig. 3.

3. SPECIMENS PREPARATION.

The method of pluvial deposition was adopted to prepare specimens both for the calibration chamber and for triaxial tests. This method, exhaustively discussed by Jacobsen (1976) and Battaglio et al. (1979) and others, allows one to obtain specimens of very uniform density, with relative density ($D_r$) varying between 35% and 100%; moreover it leads for a given time of deposition, to specimens with well repeatable dry bulk density ($\gamma_d$).

The description of the sand spreader used to prepare specimens for the calibration chamber is given in Battaglio et al. (1979); the 1% specimens were manufactured using small laboratory sand spreaders developed by N.C.I., Battaglio et al. (1978) and ISMES; the device used is shown in fig. 4.

In the present paper two classes of sand density are considered, namely:

<table>
<thead>
<tr>
<th>Calibrating Chamber Tests</th>
<th>Triaxial Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_d$ (KN/m$^3$)</td>
<td>$D_r$ (%)</td>
</tr>
<tr>
<td>Dense sand</td>
<td>15.50</td>
</tr>
<tr>
<td>Very dense sand</td>
<td>16.03</td>
</tr>
</tbody>
</table>

4. CALIBRATION CHAMBER TESTS.

Seventeen calibration chamber tests are considered here. The tests were performed with N.C. sand specimens, using the boundary conditions BC1 and BC3, which hopefully cover the real field situation.

The results are shown in table 1; the cone point resistance $q_{cp}$ and local skin friction resistance $f_{ps}$ values given in table 1 were obtained at 75 cm penetration depth, corresponding to the midheight of the specimen, at which a well defined plateau has almost always been observed.

Table 1 reports some other relevant information obtained during the one-dimensional compression phase which precedes the penetration phase.

5. TRIAXIAL TESTS.

Triaxial tests were performed on pluvially deposited cylindrical specimens 3.82 cm in diameter and 7.64 cm in height using the stress path controlled triaxial cell [Bishop and Hasley (1976)] with the performance feedback system shown in fig. 5 [see also Menzies et al. (1979)].

Isotropically consolidated and drained compression tests (TX-CDO) were performed and the results can be summarized as follows:

5.1. Strength envelope.

For both classes of relative density the strength envelope is not linear (see fig. 6) and can be well approximated by the following function, proposed by Bagni (1975, 1976):

$$\tan \phi' + \tan \alpha = \frac{1}{2.3} \left( \frac{1 - \phi'}{\phi''} \right)$$

where:

- $c =$ cohesion intercept
- $\phi' = $ angle of friction at the reference normal stress $\sigma_0$ ($\sigma_0 = 1$ kg/cm$^2$, say)
- $\alpha = $ angle describing the curvature of the envelope; when $\alpha$ equals zero the envelope is straight.
### TABLE 1
Calibration chamber test results

<table>
<thead>
<tr>
<th>Class of Sand</th>
<th>Test</th>
<th>$D_{RI}$ (%)</th>
<th>$D_{RC}$ (%)</th>
<th>$K_{OC}$ (-)</th>
<th>$K_{op}$ (-)</th>
<th>$\sigma_{VC}$ (KN/m$^2$)</th>
<th>$N_0$ (KN/m$^2$ · 10$^2$)</th>
<th>$q_C$ (KN/m$^2$ · 10$^2$)</th>
<th>FR (%)</th>
<th>BC</th>
<th>$\phi_s^c$ / $\phi_s^c$</th>
<th>$S_R$ / $I_R$</th>
<th>$z_{eq}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VERY DENSE SAND</strong></td>
<td>19</td>
<td>90.1</td>
<td>92.9</td>
<td>0.423</td>
<td>0.512</td>
<td>515.0</td>
<td>1467</td>
<td>464.5</td>
<td>0.70</td>
<td>3</td>
<td>39.7/41.4</td>
<td>105/160</td>
<td>32.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>91.8</td>
<td>93.6</td>
<td>0.409</td>
<td>0.560</td>
<td>373.9</td>
<td>1231</td>
<td>381.6</td>
<td>0.73</td>
<td>3</td>
<td>40.3/41.9</td>
<td>118/179</td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>92.9</td>
<td>93.6</td>
<td>0.390</td>
<td>0.704</td>
<td>115.8</td>
<td>851</td>
<td>239.2</td>
<td>0.63</td>
<td>3</td>
<td>41.5/42.8</td>
<td>149/225</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>91.8</td>
<td>93.6</td>
<td>0.405</td>
<td>0.530</td>
<td>312.9</td>
<td>1261</td>
<td>361.8</td>
<td>0.54</td>
<td>3</td>
<td>40.3/41.9</td>
<td>118/179</td>
<td>19.4</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>86.3</td>
<td>87.6</td>
<td>0.421</td>
<td>0.820</td>
<td>65.7</td>
<td>755</td>
<td>184.4</td>
<td>0.59</td>
<td>3</td>
<td>42.0/43.0</td>
<td>168/255</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>93.6</td>
<td>96.6</td>
<td>0.427</td>
<td>0.422</td>
<td>512.1</td>
<td>1444</td>
<td>437.0</td>
<td>0.92</td>
<td>3</td>
<td>39.7/41.4</td>
<td>105/160</td>
<td>32.0</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>95.5</td>
<td>96.5</td>
<td>0.404</td>
<td>0.412</td>
<td>121.6</td>
<td>874</td>
<td>209.3</td>
<td>0.58</td>
<td>3</td>
<td>41.4/42.8</td>
<td>147/222</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>89.8</td>
<td>90.5</td>
<td>0.373</td>
<td>0.373</td>
<td>68.7</td>
<td>766</td>
<td>119.6</td>
<td>0.53</td>
<td>3</td>
<td>42.0/43.0</td>
<td>166/253</td>
<td>4.3</td>
</tr>
<tr>
<td><strong>DENSE SAND</strong></td>
<td>22</td>
<td>69.0</td>
<td>71.2</td>
<td>0.423</td>
<td>0.487</td>
<td>311.0</td>
<td>1499</td>
<td>261.1</td>
<td>0.61</td>
<td>3</td>
<td>33.1/35.0</td>
<td>134/189</td>
<td>20.1</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>68.2</td>
<td>69.4</td>
<td>0.416</td>
<td>0.531</td>
<td>113.8</td>
<td>761</td>
<td>156.5</td>
<td>0.65</td>
<td>3</td>
<td>34.7/36.2</td>
<td>164/230</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>68.7</td>
<td>71.2</td>
<td>0.436</td>
<td>0.485</td>
<td>514.0</td>
<td>1365</td>
<td>344.3</td>
<td>0.56</td>
<td>3</td>
<td>32.2/34.2</td>
<td>127/171</td>
<td>33.2</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>67.5</td>
<td>71.6</td>
<td>0.442</td>
<td>0.475</td>
<td>716.1</td>
<td>1508</td>
<td>407.1</td>
<td>0.50</td>
<td>3</td>
<td>31.6/33.6</td>
<td>117/153</td>
<td>46.4</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>66.7</td>
<td>57.5</td>
<td>0.407</td>
<td>0.600</td>
<td>65.7</td>
<td>620</td>
<td>108.6</td>
<td>0.62</td>
<td>3</td>
<td>35.5/36.6</td>
<td>183/257</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>68.2</td>
<td>69.3</td>
<td>0.412</td>
<td>0.343</td>
<td>115.7</td>
<td>779</td>
<td>135.6</td>
<td>0.69</td>
<td>3</td>
<td>34.7/36.2</td>
<td>164/230</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>72.8</td>
<td>74.0</td>
<td>0.413</td>
<td>0.356</td>
<td>114.8</td>
<td>806</td>
<td>120.6</td>
<td>0.62</td>
<td>3</td>
<td>34.7/36.2</td>
<td>164/230</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>72.8</td>
<td>75.2</td>
<td>0.432</td>
<td>0.422</td>
<td>313.9</td>
<td>1148</td>
<td>221.3</td>
<td>0.60</td>
<td>3</td>
<td>33.1/35.0</td>
<td>134/189</td>
<td>20.1</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>77.2</td>
<td>80.6</td>
<td>0.445</td>
<td>0.441</td>
<td>509.1</td>
<td>1358</td>
<td>316.5</td>
<td>0.77</td>
<td>3</td>
<td>32.3/34.2</td>
<td>127/172</td>
<td>32.6</td>
</tr>
</tbody>
</table>

$D_{RI}$ = initial relative density  
$D_{RC}$ = relative density after consolidation  
$K_{OC}$ = ratio $\frac{\sigma_0}{\sigma_0'}$ at the end of consolidation  
$K_{op}$ = ratio $\frac{\sigma_0}{\alpha_{VC}}$ during the penetration  
$\sigma_{VC}$ = vertical consolidation stress  
$N_0$ = constrained modulus at $\sigma_{VC}$  
$q_C$ = cone resistance  

BC = boundary conditions during penetration test  
$S_R$, $I_R$ = triaxial rigidity index  
$\phi_s^c$, $\phi_s^c$ = $\phi_s$ values obtained from expanding cavity theory with non-linear strength envelope  
SUFFIX: C = Cylindrical; S = Spherical  
$z_{eq}$ = equivalent depth, obtained from the ratio $\frac{\sigma_{VC}}{\gamma_d}$  
($\gamma_d$ = initial dry density)  
FR = friction ratio

5.2. Young's Modulus.  
As far as Young's modulus ($E'$) are concerned, they have been evaluated at stress level half the stress at failure ($E_{50}'$); from the experimental results we obtained the following relationships between $E_{50}'$ and $\sigma_0$ (effective consolidation stress).

**Dense sand:**  
$$E_{50}' = 41155 \left(\frac{\sigma_0}{\sigma_0'}\right) 0.7304 \text{ kN/m}^2 \quad (2)$$

**Very dense sand:**  
$$E_{50}' = 46840 \left(\frac{\sigma_0}{\sigma_0'}\right) 0.7215 \text{ kN/m}^2 \quad (3)$$

$\sigma_0'$ = reference stress at $\sigma_0' = 100 \text{ kN/m}^2$
6. THEORETICAL $q_C$

To evaluate $q_c$ one has to refer to available computation procedures which are based on the classical theory of the plasticity in a rigid-plastic body or on the theory of expanding cavities of an elastic-perfectly plastic material; this latter allows one to take into account, in an approximate way, soil deformability in both elastic and plastic zones [see Cassan (1969), Vesic (1975), Al Awadi (1975) and others].

Among the numerous computation procedures available, on the basis of the preliminary calculation made by Oonassero (1980),

- Durgunoglu and Mitchell's procedure (1973, 1975), as an example of classical bearing capacity theory.
- Vesic's approaches (1975, 1977), related to cylindrical and spherical expanding cavities.

Durgunoglu and Mitchell (1975) proposed the following expressions for the evaluation of cone resistance in sand:

$$q_c = pgB N_{q_k} \gamma_q$$  \hspace{1cm} (6)

where:

$\rho$ = mass density

$g$ = acceleration of gravity

$B$ = width of penetrometer tip

$N_{q_k}$ = bearing capacity factor (on the basis of eq. (8) given in Durgunoglu and Mitchell's paper (1975))

$\gamma_q$ = shape factor (eq. (16), Durgunoglu and Mitchell)

Vesic's approach based on the theory of the cylindrical expanding cavity leads to the following approximate formula for $q_c$, when considering cohesionless material having curved strength envelopes:

$$q_c = \frac{N_{q_k}}{B} \lambda \left[ 1 + \tan \left( \frac{\pi - \phi'_s}{2} \right) \tan \phi'_s \right] \exp \left( \frac{B}{2} - \phi'_s \right)$$  \hspace{1cm} (7)

where:

$p_u^d$ = ultimate pressure of the expanding cylindrical cavity in an elasto-plastic infinite medium

$\lambda$ = empirical shape factor $= 1 + \tan \phi'_s$ (Vesic, 1974)

$\phi'_s$ = secant angle of friction, related to the average effective stress in failure zone at failure

The corresponding equation for spherical expanding cavity is (Vesic, 1977):

$$q_c = \frac{p_u^d}{\tan^2 \left( \frac{\phi'_s}{2} + \phi'_s \right)} \exp \left( \frac{B}{\lambda \tan \phi'_s} \right)$$  \hspace{1cm} (8)

where:

$p_u^d$ = ultimate pressure of the expanding spherical cavity in an elasto-plastic infinite medium

$p_u^c$ and $p_u^d$ were evaluated eqns. (7) and (8) using the theory proposed by Baligh (a non-linear strength envelope is considered). Computations were carried out by means of the computer program EXPAND developed at the Civil Eng. Dept. of M.I.T. (see Baligh (1975)). Because of inherent difficulties in the assessment of the $\phi'_s$ values to be used in the formulae, as a first approximation, it was assumed $\phi'_s$ to be close to the average mobilized $\phi$ within the plastic zone existing at failure around an expanded cavity. This was computed evaluating average shear $(\tau_f)$ and normal $(\sigma_{eff})$ stresses on the failure plane at failure for each of the soil elements the plastic zone was subdivided.
into by code EXPAND, obtaining therefore:

\[ \phi'_s = \arctan \left( \frac{\phi'_{eff}}{\phi'} \right) \text{ average} \]

7. **MEASURED \((q_c^M)\) vs. COMPUTED \((q_c^C)\) CONE RESISTANCE.**

Figs. 7 and 8 show the \(q_c\) values measured in the calibration chamber compared with those computed on the basis of the theoretical approaches mentioned in the previous paragraph, in which consistent and reliable soil parameters have been introduced.

The results allow the following remarks:

a) For very dense sand (fig. 7) both the formulae proposed by Vesic (1975, 1977) (with \(p_e\) evaluated considering a non-linear strength envelope) fit the experimental results reasonably well; this fit appears to be a little better for the cylindrical rather than for the spherical cavity approach.

For dense sand (fig. 8) the comparison between \(q_c^M\) and \(q_c^C\) is seen to be less satisfactory with respect to the results obtained for very dense sand (the experimental parameters describing the non-linear strength envelope of dense sand are thought to be less reliable than those determined for very dense sand); however Vesic's formulae, combined with appropriate input parameters, are able to predict the range of \(q_c\) with reasonable accuracy.

b) As far as Durgunoglu and Mitchell's approach is concerned, used here in connection with the angle of shearing resistance obtained linearizing (see, e.g. fig. 6) the strength envelope, fig. 7 (very dense sand) shows that it underestimates to some extent \(q_c\) at shallow depth (+) and largely overestimates \(q_c\) at depths below 20 to 25 metres. In fig. 8 (dense sand) Durgunoglu and Mitchell's approach was utilized with two different angles of shearing resistance obtained linearizing two available strength envelopes obtained from CID-Tx tests and CK-Tx tests; in this case the agreement seems to be better, but one gets the impression, as far as this method is concerned, that its use with constant \(\phi'_s\), i.e. neglecting the curvature of the strength envelope, makes it impossible to provide a reasonable assessment of \(q_c\).

This fact raises a very practical question: is it reason-

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**Fig. 7** - VERY DENSE SAND

**Fig. 8** - DENSE SAND

**Fig. 9** - Measured cone resistance vs triaxial rigidity index.

(+) but always well beyond the critical depth as defined by Durgunoglu and Mitchell, 1975.
- For Dutch CPT tip:
  \[ I_r = \frac{300}{FR} \]  
  \[ \ldots(9) \]

- For cylindrical electrical tip:
  \[ I_r = \frac{170}{FR} \]  
  \[ \ldots(10) \]

where \( FR = \frac{f_s}{q_c} \) = friction ratio (\%).

Examining the \( I_r \) and \( FR \) values given in Table 1, eq. (10), for the type of sand used in these tests, tends to overestimate \( I_r^s \) (rigidity index for spherical cavity) and to give the upper range of the experimental values of \( I_r^c \) (rigidity index for cylindrical cavity).

AKNOWLEDGEMENTS.

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NOTES:

ENEL CRIS = Hydraulic and Structural Research Center of the State and Research Department of the Italian National Electricity Board - Milano.

ISMES = Istituto Sperimentale Modelli e Strutture - Bergamo.

MIT = Massachusetts Institute of Technology - Cambridge.

NIG = Norwegian Geotechnical Institute - Oslo.

REFERENCES.


