Liquefaction Resistance of Soils from Shear-Wave Velocity

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ABSTRACT: A simplified procedure using shear-wave velocity measurements for evaluating the liquefaction resistance of soils is presented. The procedure was developed in cooperation with industry, researchers, and practitioners and evolved from workshops in 1996 and 1998. It follows the general format of the Seed-Idriss simplified procedure based on standard penetration test blow count and was developed using case history data from 26 earthquakes and >70 measurement sites in soils ranging from fine sand to sandy gravel with cobbles to profiles including silty clay layers. Liquefaction resistance curves were established by applying a modified relationship between the shear-wave velocity and cyclic stress ratio for the constant average cyclic shear strain suggested by R. Dobry. These curves correctly predicted moderate to high liquefaction potential for ≥95% of the liquefaction case histories and are shown to be consistent with the standard penetration test based curves in sandy soils. A case study is provided to illustrate application of the procedure. Additional data are needed, particularly from denser soil deposits shaken by stronger ground motions, to further validate the simplified procedure.

INTRODUCTION

Evaluation of the liquefaction resistance of soils is an important step in many geotechnical investigations in earthquake-prone regions. The procedure widely used in the United States and throughout much of the world for evaluating soil liquefaction resistance is termed the “simplified procedure.” This simplified procedure was originally developed by Seed and Idriss (1971) using blow counts from the standard penetration test (SPT) correlated with a parameter called the cyclic stress ratio that represents the cyclic loading on the soil. Since 1971, this procedure has been revised and updated (Seed 1979; Seed and Idriss 1982; Seed et al. 1983, 1985; Youd et al. 1997). In the mid-1980s, a parallel procedure based on the cone penetration test (CPT) was introduced by Robertson and Campagnola (1985), which also has been revised and updated (Seed and de Alba 1986; Stark and Olson 1993; Olsen 1997; Robertson and Wride 1998).

A promising alternative, or supplement, to the penetration-based approach is provided by in situ measurements of small-strain shear-wave velocity \( V_s \). The use of \( V_s \) as an index of liquefaction resistance is soundly based because both \( V_s \) and liquefaction resistance are similarly influenced by many of the same factors (e.g., void ratio, state of stress, stress history, and geologic age). Some advantages of using \( V_s \) are that (Dobry et al. 1981; Seed et al. 1983; Stokoe et al. 1988; Tokimatsu and Uchida 1990) (1) the measurements are possible in soils that are hard to sample, such as gravelly soils where penetration tests may be unreliable; (2) measurements can also be performed on small laboratory specimens, allowing direct comparisons between laboratory and field behavior; (3) \( V_s \) is a basic mechanical property of soil materials, directly related to small-strain shear modulus \( G_{max} \) by

\[
G_{max} = \rho V_s^2
\]

where \( \rho \) = mass density of soil; (4) \( G_{max} \), or \( V_s \), is normally a required property in earthquake site response and soil-structure interaction analyses; and (5) \( V_s \) can be measured by the spectral-analysis-of-surface-waves (SASW) technique at sites where borings may not be permitted, such as capped landfills, sites that extend for great distances where rapid evaluation is required, and hard-to-sample sites composed of gravels, cobbles, and even boulders.

Three concerns when using \( V_s \) to evaluate liquefaction resistance are that (1) no samples are routinely obtained as part of the testing procedure for soil classification and identification of nonliquefiable materials; (2) thin, low \( V_s \) strata may not be detected if the measurement interval is too large (U.S. Bureau of Reclamation (USBR) 1989; Boulanger et al. 1997); and (3) measurements are made at small strains, whereas pore-water pressure buildup and liquefaction are medium- to high-strain phenomena (Jamiolkowski and Lo Presti 1990; Teachavorasiskun et al. 1994; Roy et al. 1996). This third concern can be significant for cemented soils, because small-strain measurements are highly sensitive to weak interparticle bonding that is eliminated at medium and high strains. It also can be significant in silty soils above the water table where negative pore-water pressures can increase \( V_s \).

Over the past 20 years, numerous studies have been conducted to investigate the relationship between \( V_s \) and liquefaction resistance. These studies involved field performance observations [e.g., Stokoe and Nazarian (1985), Robertson et al. (1992), Kayen et al. (1992), and Andrus and Stokoe (1997)], penetration-\( V_s \) correlations [e.g., Seed et al. (1983) and Lodge (1994)], analytical investigations [e.g., Bierschewke and Stokoe (1984) and Stokoe et al. (1988b)], and laboratory tests [e.g., Dobry et al. (1981), de Alba et al. (1984), and Tokimatsu and Uchida (1990)]. Several of the liquefaction evaluation procedures developed from these studies follow the general format of the Seed-Idriss simplified procedure, where \( V_s \) is corrected to a reference overburden stress and correlated with the cyclic stress ratio. Nearly all were developed with limited or no field performance data.

Summarized in this paper is the procedure originally proposed in the workshop paper by Andrus and Stokoe (1997) and subsequently updated in the project report by Andrus et al. (1999). The procedure is based on field performance data from 26 earthquakes and in situ \( V_s \) measurements from >70 sites. Suggestions from two technical workshops have been incorporated into the procedure. The first workshop was held on January 4–5, 1996, in Salt Lake City, and was sponsored by the National Center for Earthquake Engineering Research (NCEER). The second workshop was held on August 14–15, 1998, also in Salt Lake City, and was sponsored by the Multidisciplinary Center for Earthquake Engineering Research.
EVALUATION PROCEDURE

The evaluation procedure requires the calculation of three parameters: (1) the level of cyclic loading on the soil caused by the earthquake, expressed as a cyclic stress ratio; (2) stiffness of the soil, expressed as an overburden stress-corrected shear-wave velocity; and (3) resistance of the soil to liquefaction, expressed as a cyclic resistance ratio. Each parameter is discussed below.

Cyclic Stress Ratio CSR

The cyclic stress ratio, $\tau_{cr}/\sigma'_{v0}$, at a particular depth in a level soil deposit can be expressed (Seed and Idriss 1971):

$$CSR = \frac{\tau_{cr}}{\sigma'_{v0}} = 0.65 \left( \frac{a_{\text{max}}}{g} \right) \frac{\sigma'_{v}}{\sigma'_{v0}} r_d$$  (2)

where $\tau_{cr}$ = average equivalent uniform cyclic shear stress caused by the earthquake and is assumed to be 0.65 of the maximum induced stress; $a_{\text{max}}$ = peak horizontal ground surface acceleration; $g$ = acceleration of gravity; $\sigma'_{v}$ = initial effective vertical (overburden) stress at the depth in question; $\sigma'_{v0}$ = total overburden stress at the same depth; and $r_d$ = shear stress reduction coefficient to adjust for the flexibility of the soil profile.

Values of $r_d$ are commonly estimated from the chart by Seed and Idriss (1971), using the average curve shown in Fig. 1. Their average curve was determined analytically using a variety of earthquake motions and soil conditions. Revised average $r_d$ values have been proposed by Idriss (1999) based on the analytical work by Golewslicki (1989). Unlike the original $r_d$ values, these revised $r_d$ values are magnitude dependent. As shown in Fig. 1, the revised $r_d$ curve for moment magnitude $M_w = 7.5$ is almost identical to the average curve published by Seed and Idriss (1971).

Stress-Corrected Shear-Wave Velocity

Shear-wave velocities can be measured in situ by several seismic tests including cross hole, downhole, seismic cone penetrometer, suspension logger, and SASW. A review of these test methods is given in Woods (1994). Their accuracy can be sensitive to procedural details, soil conditions, and interpretation techniques.

One important factor influencing $V_s$ is the state of stress in soil (Hardin and Drnevich 1972). Laboratory test results (Roesler 1979; Stokoe et al. 1985; Belloti et al. 1996) show that the velocity of a propagating shear wave depends equally on principal stresses in the direction of wave propagation and particle motion. Thus, $V_s$ measurements made with wave propagation or particle motion in the vertical direction can be related by the following empirical relationship:

$$V_s = A(\sigma'_{h0})^{m}(\sigma'_{v0})^{m}$$  (3)

where $A$ = parameter that depends on the soil structure; $\sigma'_{h0}$ = initial effective horizontal stress at the depth in question; and $m$ = stress exponent with a value of about 0.125.

Following the traditional procedures for correcting SPT blow count and CPT tip resistances to account for overburden stress, one can correct $V_s$ to a reference overburden stress by (Sykora 1987; Robertson et al. 1992)

$$V_{s1} = V_s C_v = V_s \left( \frac{P_o}{\sigma'_{v0}} \right)^{0.25}$$  (4)

where $V_{s1}$ = overburden stress-corrected shear-wave velocity; $C_v$ = factor to correct measured shear-wave velocity for overburden pressure; $P_o$ = reference stress of 100 kPa or about atmospheric pressure; and $\sigma'_{v0}$ = initial effective overburden stress (kPa). A maximum $C_v$ value of 1.4 is generally applied to $V_s$ data at shallow depths, similar to the SPT and CPT procedures. In using (4), it is implicitly assumed from the relationship given in (3) that the initial effective horizontal stress $\sigma'_{h0}$ is a constant factor of the effective overburden stress. The factor, generally referred to as $K_h$, is assumed to be approximately 0.5 at natural, level-ground sites where liquefaction has occurred or is likely to occur. Also, in applying (4), it is implicitly assumed that $V_s$ is measured with both particle motion and wave propagation polarized along principal stress directions and one of those directions is vertical.

Cyclic Resistance Ratio CRR

The value of CSR separating liquefaction and nonliquefaction occurrences for a given $V_{s1}$, or corrected penetration resistance, is called the cyclic resistance ratio CRR. R. Dobry (personal communication, January 6, 1996) derived a relationship between CSR and $V_{s1}$ for a constant average cyclic shear strain using (1) and

$$\gamma_{cr} = \frac{\tau_{cr}}{(G_{sec})}$$  (5)

where $\gamma_{cr}$ = average peak shear strain during a cyclic stress-controlled test of uniform cyclic shear stress $\tau_{cr}$; and $G_{sec}$ = secant shear modulus at $\gamma_{cr}$ during the same cyclic test. By combining (1) and (5), the following relationship is obtained:

$$CSR = \frac{\tau_{cr}}{\sigma'_{v0}} = f(\gamma_{cr}) V_{s1}$$  (6)

where $f(\gamma_{cr})$ = function of $\gamma_{cr}$. Because CSR equals CRR at the point separating liquefaction and nonliquefaction, (6) provides an analytical basis for establishing the CRR-$V_{s1}$ curves at low values of $V_{s1}$ (say $V_{s1} \leq 125$ m/s) and extending them to zero at $V_{s1} = 0$.

Andrus and Stokoe (1997) modified (6) to

$$CRR = \left( a \left( \frac{V_{s1}}{100} \right)^2 + b \left( \frac{1}{V_{s1}^{0.5}} - \frac{1}{V_{s1}^{0.5}} \right) \right) \text{MSF}$$  (7)

where $V_{s1}^{0.5}$ = limiting upper value of $V_{s1}$ for cyclic liquefaction occurrence; $a$ and $b$ = curve fitting parameters; and MSF = magnitude scaling factor to account for the effect of earthquake magnitude. The first term in (7) is a form of (6), assuming $f(\gamma_{cr})$ is independent of initial effective confining pressure.
and pore-water pressure buildup. The second term is a hyperbola with a small value at low values of \( V_{51} \) and a very large value as \( V_{51} \) approaches \( V_{51}^* \).

The assumption of a limiting upper value of \( V_{51} \) is equivalent to the assumption commonly made in the SPT- and CPT-based procedures dealing with clean sands, where liquefaction is considered not possible above a corrected blow count of about 30 (Seed et al. 1985) and corrected tip resistance of about 160 (Robertson and Wride 1998). Upper limits for \( V_{51} \) and penetration resistance are explained by the tendency of dense soils to exhibit dilative behavior at large strains, causing negative pore-water pressures. Although it is possible in a dense soil to generate pore-water pressures close to the confining stress if large cyclic strains or many cycles are applied, the amount of water expelled during reconsolidation is dramatically less for dense soils than for loose soils. As explained by Dobry (1989), in dense soils, settlement is insignificant and no sand boils or failure take place because of the small amount of water expelled. This is important because the definition of liquefaction used to classify the field behavior here, as well as in the penetration-based procedures, is based on surface manifestations.

The magnitude scaling factor is traditionally applied to CRR, rather than the cyclic loading parameter CSR, and equals 1.0 for earthquakes with a magnitude of 7.5. For magnitudes other than 7.5, Fig. 2 presents magnitude scaling factors developed by various investigators. The 1996 NCEER workshop (Youd et al. 1997) recommended a range of factors that can be represented by

\[
MSF = \left( \frac{M_r}{7.5} \right)^n
\]

where \( M_r = \) moment magnitude; and \( n \) = exponent. Moment magnitude is the scale most commonly used for engineering applications and is preferred for liquefaction resistance calculations (Youd et al. 1997). The lower bound for the range of MSFs recommended by the 1996 NCEER workshop is defined with \( n = -2.56 \) (I. M. Idriss, personal communication, October, 1995). The upper bound of the recommended range is defined with \( n = -3.3 \) (Andrus and Stokoe 1997) for earthquakes with magnitudes \( \leq 7.5 \). Magnitude scaling factors defined by (8) and average \( r_j \) values originally proposed by Seed and Idriss (1971) should be used together when applying (2) and (7).

More recently, Idriss (1999) proposed revised MSFs defined by

\[
MSF = 6.9 \exp \left( -\frac{M_r}{4} \right) - 0.06, \text{ for } M_r > 5.2 \tag{9a}
\]

\[
MSF = 1.82, \text{ for } M_r \leq 5.2 \tag{9b}
\]

Magnitude scaling factors defined by (9) and revised \( r_j \) proposed by Idriss (1999) should be used together when applying (2) and (7). The difference in the two proposed MSF and \( r_j \) relationships is not significant for earthquakes with magnitudes of about 7–7.5 (Andrus et al. 1999), the range of the majority of the \( V_{51} \) case history data.

### CASE HISTORY DATA

Shear-wave velocity measurements have been performed at many liquefaction sites. A summary of >70 sites (139 test arrays) and 26 earthquakes that have been studied by various investigators is given in Andrus et al. (1999). Table 1 presents a list of these 26 earthquakes, with the soil types associated with each case history. Pertinent characteristics of the case history data are described in more detail below.

Andrus et al. (1999) defined a case history as an earthquake and a test array. A test array is defined as the two boreholes used for cross-hole measurements, the borehole and source used for downhole measurements, the cone sounding and source used for seismic cone measurements, the borehole used for suspension logger measurements, or the line of receivers used for SASW measurements. By combining the 139 test arrays and 26 earthquakes, a total of 225 case histories were obtained, with 149 from the United States, 36 from Taiwan, 34 from Japan, and 6 from China.

The distribution of case histories with earthquake magnitude and liquefaction occurrence is presented in Fig. 3(a). When magnitude scales other than \( M_r \) were reported, they were converted to \( M_r \) using the relationship adopted by the 1996 NCEER workshop (Youd et al. 1997). The occurrence of li-

### TABLE 1. Earthquakes Used to Establish CRR-\( V_{51} \) Curves

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>( M_r )</th>
<th>Sands and Silts</th>
<th>Gravels</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>1906 San Francisco, Calif.</td>
<td>7.7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1957 Daly City, Calif.</td>
<td>5.3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1964 Niigata, Japan</td>
<td>7.5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1975 Hachinomori, China</td>
<td>7.3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1979 Imperial Valley, Calif.</td>
<td>6.5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1980 Chiba-Isaragi, Japan</td>
<td>5.9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1981 Westmorland, Calif.</td>
<td>5.9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1983 Borah Peak, Idaho</td>
<td>6.9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1985 Chiba-Isaragi, Japan</td>
<td>6.9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1986 Event LSST3, Taiwan</td>
<td>5.3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST3, Taiwan</td>
<td>5.5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST4, Taiwan</td>
<td>5.6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST5, Taiwan</td>
<td>5.4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST6, Taiwan</td>
<td>5.3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST7, Taiwan</td>
<td>6.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST8, Taiwan</td>
<td>6.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST12, Taiwan</td>
<td>6.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST13, Taiwan</td>
<td>6.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1986 Event LSST16, Taiwan</td>
<td>7.6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1987 Chiba-toho-o-ko, Japan</td>
<td>6.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1987 Elmore Ranch, Calif.</td>
<td>5.9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1987 Superstition Hills, Calif.</td>
<td>6.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1989 Loma Prieta, Calif.</td>
<td>7.0</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>1993 Kushiro-oki, Japan</td>
<td>8.3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1993 Hokkaido-nansei, Japan</td>
<td>8.3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1994 Northridge, Calif.</td>
<td>8.3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1995 Hyogo-ken Nanbu, Japan</td>
<td>8.9</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 2.** Magnitude Scaling Factors Derived by Various Investigators and Range Recommended by 1996 NCEER Workshop [Modified from Youd et al. (1997)]
Liquefaction was based on the appearance of surface evidence, such as sand boils, ground cracks and fissures, and ground settlement. At five sites the assessment of liquefaction or non-liquefaction occurrence was supported by pore-water pressure measurements. In addition, liquefaction occurrence was assigned (in this paper) to the Treasure Island, Calif., fire station cases, where the strong ground motion records from the 1989 Loma Prieta earthquake exhibit a sudden drop at about 15 s and small motion afterward (Idriss 1990), indicating liquefaction (de Alba et al. 1994). Of the 225 case histories, 99 were liquefaction case histories and 126 were nonliquefaction case histories.

Values of $V_s$ reported by the investigators were directly reported. Depending on the test method, in situ $V_s$ measurements may be reported at discrete depths or for continuous intervals. When velocities were reported for continuous intervals, as is typically the case for downhole, seismic cone, suspension logger, and SASW measurements, the depth to the center of each interval was assumed. Thus, if the reported $V_s$ profile had 10 velocity layers, it was assumed that the profile consisted of 10 “measurements” with depths at the center of each layer. Only the cross-hole measurements made with shear waves having particle motion in the vertical direction were used. Cross-hole measurements near the critical layer boundary that seemed high, and could represent refracted waves, were not included in the average. Some $V_s$ values were from measurements performed before the earthquake, others followed the earthquake. No adjustments were made to compensate for changes in soil density and $V_s$ due to ground shaking.

The layer of soil most likely to liquefy at a site, or the critical layer, was the layer of nonplastic soil below the ground-water table where values of $V_{s3}$ and penetration resistance were generally the least and CSR relative to $V_{s3}$ was the greatest. In Fig. 3(b), the distribution of case histories with earthquake magnitude, predominate soil type (gravel, sand, or silt) and average fines content (silt and clay) is presented. Of the 225 case histories, 28 were for sands with fines content $FC \leq 5\%$, 90 for sands with $FC = 6\% - 34\%$, 71 for sands and silts with $FC \approx 35\%$, 26 for gravels with $FC \leq 5\%$, and 10 for gravels with $FC = 6\% - 34\%$. From the cumulative relative frequency distributions presented in Fig. 4, about 90% of the case histories had a critical layer thickness $<7$ m, average measurement depth $<8$ m, and water table depth $<4$ m. Overall, the gravel case histories exhibit smaller layer thicknesses and shallower measurement depths than do the sand and silt case histories.

About 70% of the case histories were for natural soil deposits, with many formed by alluvial processes. The other 30% were for hydraulic or dumped fills. Eight of the fills had been densified by soil improvement techniques. At least 85% of the case histories were for soils of Holocene age ($<10,000$ years). Although the ages of the other 15% were unknown, they were believed to be also of Holocene age.

Values of $\sigma_r$ and $\sigma_r'$ were estimated using soil densities reported by the investigators. When no densities were reported, typical values for soils with similar grain size, penetration, and velocity characteristics were assumed. In most instances, the assumed densities were 1.76 Mg/m$^3$ for soils above the water table and 1.92 Mg/m$^3$ for soils below the water table.

Because many published attenuation relationships between $a_{\text{max}}$ and source distance are based on peak acceleration values obtained from ground motion records for two horizontal directions (sometimes referred to as the randomly oriented horizontal component), the geometric mean (square root of the product) of the two peak values was used. Use of the geometric mean is consistent with the development of the SPT-based procedure (Yosh et al. 1997).

Values of $V_{s1}$ and CSR were first calculated for each measurement depth within the critical layer and then averaged. In the calculations, each site was assumed to be level ground. Values of $C_s$ used to correct measured shear-wave velocities ranged from 1.4 to 0.9 for most of the data. About 80% of the case histories have two to seven values on average.

**LIQUEFACTION EVALUATION CHARTS**

In the process of developing the liquefaction evaluation charts, all case history data were initially plotted on the same chart. This aggregation was accomplished through an adjustment procedure; that is, the CSR values in each case history were adjusted to an earthquake with $M_w = 7.5$ by dividing by (8) with $n = -2.56$. As done in penetration evaluation procedures, the sandy soil case histories were separated into three categories: (1) Sands with average $FC \leq 5\%$; (2) sands with average $FC = 6\% - 34\%$; and (3) sands and silts with average $FC \geq 35\%$. For consistency, the gravelly soil case histories also were divided into the same three categories based on fines
content. However, no case histories exist in the database with gravel having FC \( \geq 5\% \). All data are plotted in Fig. 5 along with the proposed CRR-\( V_{S1} \) curves. Development of these curves is discussed below.

**Limiting Upper Value of \( V_{S1} \) in Sandy Soils**

As shown in Fig. 5, CSR values above about 0.35 are limited in the case history data. Thus, current estimates of \( V_{S1}^{*} \) rely, in part, on penetration-\( V_{S} \) correlations and, in part, on the data trend in Fig. 5. Furthermore, the penetration-\( V_{S} \) correlations are strongly biased toward measurements in sandy soils, because these types of measurements on gravelly and cobbly soils are still in the early application stages.

In the SPT-based procedure, a corrected blow count \((N)_{ho}^{*}\) of 30 is assumed as the limiting upper value for cyclic liquefaction occurrence in sands with \( \leq 5\% \) silt and clay (Seed et al. 1985). The correlation by Ohta and Goto (1978) modified to a blow count with a theoretical free-fall energy of 60% (Seed et al. 1985) suggested equivalent \( V_{S1}^{*} \) values of 207 m/s for Holocene sands, assuming that a depth of 10 m is equivalent to an effective overburden stress of 100 kPa. The stress-corrected cross-hole measurements compiled by Sykora (1987) for Holocene sands and nonplastic silty sands below the ground-water table, with \((N)_{ho}^{*}\) between 25 and 35, exhibit an average \( V_{S1}^{*} \) value of 206 m/s and standard deviation of 41 m/s. Finally, the case history data in this study were used to investigate the \( V_{S1}^{*} \) and \((N)_{ho}^{*} \) relationship for well-documented sand layers with \( \leq 10\% \) fines. These data are presented in Fig. 6 along with the best-fit relationship that can be expressed

\[
V_{S1}^{*} = B_{1}[(N)_{ho}^{*}]^{b_{1}}
\]

(10)

where \( B_{1} = 93.2 \pm 6.5 \) and \( b_{1} = 0.231 \pm 0.022 \) for soils with fines content \( \leq 10\% \) and with \( V_{S1}^{*} \) in meters per second and \((N)_{ho}^{*}\) in blows/0.3 m. The plotted data exhibit a mean \( V_{S1}^{*} \) value of 204 m/s at a \((N)_{ho}^{*}\) value of 30 and residual standard deviation \( S_{res} \) of 12 m/s.

From these estimates, a \( V_{S1}^{*} \) value of 210 m/s is assumed equivalent to a \((N)_{ho}^{*}\) value of 30 in clean sands (\( \leq 5\% \) fines).

A value of 210 m/s for cyclic liquefaction occurrence at CSR = 0.6 is less than the general consensus value of 230 m/s suggested at the 1998 NCEER workshop. As a result, Fig. 6 was added specifically to provide additional evidence to support the use of 210 m/s in clean sands.

For sandy soils with FC \( \geq 35\% \), the SPT-based chart by Seed et al. (1985) indicated a limiting upper \((N)_{ho}^{*}\) value of about 21 for cyclic liquefaction occurrence. The correlation by Ohta and Goto (1978) suggested equivalent \( V_{S1}^{*} \) values of 195 m/s for Holocene sands. The stress-corrected cross-hole measurements compiled by Sykora (1987) for Holocene sands and nonplastic silty sands below the ground-water table with \((N)_{ho}^{*}\) between 16 and 26 exhibited an average value of 190 m/s and standard deviation of 36 m/s. From these estimates, a \( V_{S1}^{*} \) value of 195 m/s is assumed equivalent to an \((N)_{ho}^{*}\) value of 21 in soils with FC \( \geq 35\% \).

To permit the CRR-\( V_{S1} \) curves for magnitude 7.5 earthquakes to have \( V_{S1}^{*} \) values between 195 and 210 m/s at CRR near 0.6, values of \( V_{S1}^{*} \) are assumed to range linearly from 200 to 215 m/s. The relationship between \( V_{S1}^{*} \) and fines content can be expressed by

\[
V_{S1}^{*} = 215 - 0.5(FC - 5) \text{ m/s}, \quad \text{for sands with } 5\% < FC < 35\%
\]

(11b)

\[
V_{S1}^{*} = 200 \text{ m/s}, \quad \text{for sands and silts with } FC \geq 35\%
\]

(11c)

where FC = average fines content in percent by mass.

To illustrate how well the recommended CRR-\( V_{S1} \) curves defined by (7) and (11) fit the case history data, the data, separated by soil type, are presented in Figs. 7(a–d). The recommended curves provide reasonable bounds for all case history data above a CSR value of 0.35, indicating the use of the suggested \( V_{S1}^{*} \) values for sands and silts, as well as gravels. The use of these \( V_{S1}^{*} \) values for gravels is discussed below.

**Curve-Fitting Parameters \( a \) and \( b \)**

The three CRR-\( V_{S1} \) curves shown in Figs. 5 and 7 were determined through an iterative process of varying the values of \( a \) and \( b \) until nearly all case histories were bounded by the curves with the least amount of nonliquefaction case histories in the liquefaction region. The final values of \( a \) and \( b \) used to draw the curves were 0.022 and 2.8, respectively.

Of the 99 liquefaction case histories shown in Figs. 5 and
FIG. 7. Curves Recommended for Calculation of CRR from $V_s$, Measurements in Sands and Gravels along with Case History Data Separated by Soil Type

7, only two incorrectly lie in the No Liquefaction region. The two case histories that incorrectly lie in the No Liquefaction region are two sites at Treasure Island, Calif., where liquefaction was marginal during the 1989 Loma Prieta earthquake ($M_w = 7$). It is interesting to note that similar incorrect evaluations also are obtained when one uses the SPT data for these two sites (Andrus et al. 1999).

To illustrate the effect of using different values of $MSF$ and $r_{irr}$, the values of $CSR$ for the case history data have been recalculated using the revised values of $MSF$ and $r_{irr}$ proposed by Idriss (1999). The recalculated case history data are plotted in Fig. 8. Also plotted in Fig. 8 are the same three CRR-$V_s$ curves shown in Fig. 5. Many of the case history data in Fig. 8 plot at higher CSR values than in Fig. 5, because the earthquake magnitude is $\leq 7.5$ for most of the data. The upward shift in the liquefaction data points near CSR of 0.1 is $< 0.01$. This difference is not significant and is within the accuracy of the plotted data.

At magnitudes less than about 7, the difference in using values of $MSF$ and $r_{irr}$ proposed by Idriss (1999) and those adopted by the NCEER workshop (Youd et al. 1997) is significant in the calculation of CSR. For $M_w$ near 5.5, the differences in CSR are about 0.02 at $V_s = 100$ m/s and 0.1 at $V_s = 200$ m/s.

Limiting Upper Value of $V_s$, in Gravelly Soils

Although the $V_s^m$ values given in (11) were determined for sandy soils, the results presented in Fig. 7(d) indicate that these limits also represent reasonable limits for gravelly soils divided into the same categories based on fines content. This might be considered rather surprising based on the penetration $V_s$ correlations presented in the literature for gravelly soils. For instance, the correlation by Ohta and Goto (1978) suggested a $V_s^m$ value of 227 m/s for Holocene gravels at an equivalent $(N_{10})_{so}$ of 30. Similarly, the correlation by Rollins et al. (1998) provided a best-fit $V_s^m$ value of 232 m/s for Holocene gravels. On the other hand, all the liquefaction case history data shown in Figs. 5 and 7 exhibit $V_s^m$ values of about 200 m/s or less, suggesting that 230 m/s may be inappropriately high. To investigate further the value of $V_s^m$ in gravelly soils, laboratory studies involving $V_s$ measurements in gravelly
soils were reviewed. Kokusho et al. (1995) clearly showed that the shear-wave velocity of gravelly soils varies greatly and is highly dependent on the particle gradation. Weston (1996) showed similar results for coarse sands with gravels. In both cases, the results show that increasing the uniformity coefficient can significantly increase the shear-wave velocity in medium-density to dense gravels. On the other hand, very loose gravelly soils, even well-graded gravels, can exhibit shear-wave velocities similar to those of loose sands (Kokusho et al. 1995). The case history data presented in Fig. 7(d) support the premise that gravelly soils that are loose enough to exhibit significant liquefaction effects (soils, ground cracks, etc.) have shear-wave velocities similar to loose sands. Hence, the authors recommended the boundaries developed for sandy soils as preliminary boundaries for gravelly soils. However, additional work is clearly needed to understand the relationship between $V_s$ and liquefaction resistance of gravels.

Other CRR-$V_s$ Curves

Fig. 9 compares the CRR-$V_s$ curve for clean soils proposed in this paper with six other proposed CRR-$V_s$ curves. The best-fit curve by Tokimatsu and Uchida (1990) was determined using cyclic triaxial test results for various sands with <10% fines. It has been adjusted to be consistent with procedures outlined in this paper. The more conservative lower-bound curve for Tokimatsu and Uchida’s data (determined by Andrus et al. 1999) also is shown in Fig. 9, because the other CRR-$V_s$ curves were drawn to bound liquefaction cases. The bounding curve by Robertson et al. (1992) was developed using field performance data from sites in Imperial Valley, Calif., and four other locations. To position their curve for magnitude 7.5 earthquakes, Robertson et al. used magnitude scaling factors similar to those suggested by Seed and Idriss (1982). Kayen et al. (1992) studied four sites that did and did not liquefy during the 1989 Loma Prieta earthquake. Lodge (1994) considered the same sites that Kayen et al. studied as well as a few other sites. The curve by Lodge was established by determining high or low liquefaction potential for each layer, using available SPT blow counts and the procedure of Seed et al. (1985). Values of $V_s$ and CSR were then plotted for both layer types. The bounding curves for Kayen et al. and Lodge shown in Fig. 9 have been adjusted for magnitude 7.5 earthquakes by assuming a $MSF$ of 1.19, the lower-bound value for magnitude 7 earthquakes recommended by the 1996 NCEER workshop (Youd et al. 1997). The curve by Andrus and Stokoe (1997) was developed for the 1996 NCEER workshop, using case histories from 20 earthquakes.

As discussed by Andrus et al. (1999), many of the differences among the seven curves shown in Fig. 9 can be explained by the different levels of conservatism assumed with limited data and different methods used for selecting site variables and correction factors. Also, some errors were identified in the database by Andrus and Stokoe (1997). Thus, the CRR-$V_s$ curves proposed in this paper are recommended because they were based on the largest, most correct case history data set and procedures recommended by the 1996 NCEER workshop (Youd et al. 1997).

**Recommended CRR-$V_s$ Curves**

The recommended CRR-$V_s$ curves presented in Fig. 5 are defined by (7), (8), and (11) with $a = 0.022$, $b = 2.8$, $V_s^b = 200–215$ m/s (depending on fines content), and $n = -2.56$. The value of $-2.56$ for $n$ is recommended because it provides more conservative CRR values than $-3.3$, which is the value defining the upper bound of the range of MSFs suggested by the 1996 NCEER workshop (Youd et al. 1997) for magnitudes <7.5. Although the MSFs defined by (8) with $n = -2.56$ provide less conservative curves than the factors proposed by Idriss (1999) for magnitudes <7.5, the findings of Ambroseys (1988), I. M. Idriss (personal communication, October 1995), Arango (1996), Youd and Noble (1997), Andrus and Stokoe (1997), and Andrus et al. (1999) supported their use.

The recommended curves shown in Fig. 5 are dashed above CRR of 0.35 to indicate that field performance data are limited. They do not extend much below 100 m/s, because there are no field data to support extending them to the origin. It is important to note that these boundary curves are for extreme behavior, where boils and ground cracks occur.

**Correlation Between $V_s$ and $(N_{so})$**

One can obtain a correlation between $V_s$ and $(N_{so})$ from the recommended CRR-$V_s$ relationships and 1996 NCEER.
workshop (Youd et al. 1997) recommended SPT-based relationship by plotting values with equal CRR. Fig. 10 presents the correlation between $V_{S1}$ and $(N)_{60}$ for clean sands, based on the recommended CRR-$V_{S1}$ and CRR-$(N)_{60}$ relationships. Also shown are the field data and mean curve for sands with <10% nonplastic fines from Fig. 6. The correlation derived from the CRR relationships lies between the mean and mean +1$S_{res}$ curves. Both $V_{S1}$ and $(N)_{60}$-based liquefaction evaluation procedures provide similar predictions of liquefaction potential, when the data point lies on the CRR-based curve. When the data point plots below the CRR-based curve, the $V_{S1}$-based liquefaction evaluation procedure provides the more conservative prediction. When the data point plots above the CRR-based curve, the SPT-based procedure provides the more conservative prediction. Because most of the data points shown in Fig. 10 plot below the CRR-based curve, the $V_{S1}$-based procedure provides an overall more conservative prediction of liquefaction resistance than does the SPT-based procedure for these sites.

Factor of Safety

A common way to quantify the potential for liquefaction is in terms of a factor of safety. The factor of safety $FS$ against liquefaction can be defined by

$$FS = \frac{CRR}{CSR}$$

Liquefaction is predicted to occur when $FS \leq 1$, and liquefaction is predicted not to occur when $FS > 1$. The acceptable value of $FS$ will depend on several factors, including the acceptable level of risk for the project, potential for ground deformation, extent and accuracy of seismic measurements, availability of other site information, and conservatism in determining the design earthquake magnitude and expected value of $\alpha_{100}$.

CORRECTION FACTORS

The recommended CRR-$V_{S1}$ curves are limited to the characteristics of the database summarized earlier in this paper. Correction factors may be used to extend the curves to site conditions different from the database.

In areas of cemented and aged soils (>10,000 years), a correction factor can be added to (7) as follows:

$$CRR = \left[ a \left( \frac{K_{c}V_{S1}}{100} \right)^{2} + b \left( \frac{1}{V_{S1}^{0.5} - K_{c}V_{S1}} \right) \left( \frac{1}{V_{S1}^{0.5}} \right) \right] MSF$$

where $K_{c}$ is correction factor for high values of $V_{S1}$ caused by cementation and aging. Average estimates of $K_{c}$ for Pleistocene soils range from 0.6 to 0.8 based on penetration-$V_{S1}$ correlations (Ohta and Goto 1978; Rollins et al. 1998).

Fig. 11 illustrates a method for estimating the value of $K_{c}$ using SPT blow counts. Shown in Fig. 11(a) are the $V_{S1}-(N)_{60}$ relationships for clean and silty sands implied by the recommended CRR-$V_{S1}$ curves and 1996 NCEER workshop recommended CRR-$(N)_{60}$ curves (Youd et al. 1997). From these implied curves, the design curves shown in Fig. 11(b) were developed. In the example, the measured values of $V_{S1}$, $(N)_{60}$, and $FC$ are 220 m/s, 8, and 10%, respectively. The relationships shown in Fig. 11(b) suggest a $K_{c}$ value of 0.71 for these conditions. This method for estimating $K_{c}$ assumes that the strain level induced during penetration testing is the same strain level causing liquefaction, which may not be true because pore-water pressure build-up to liquefaction can occur at medium strains in several loading cycles (Dobry et al. 1982; Seed et al. 1983). The method also assumes that liquefaction potential and blow count are not affected by cementation, which may not be a reasonable assumption. Hence, this suggested method should be used cautiously with engineering judgment.

In soils above the ground-water table, particularly silty soils, negative pore pressures increase the effective state of stress and hence, the value of $V_{S1}$ measured in seismic tests. This effect should be considered in the estimation of $\sigma_{y}$ for correcting $V_{S1}$ to $V_{S1}$ and for computing CSR using (2).
CASE STUDY

To illustrate the evaluation procedure, the liquefaction potential at the Treasure Island fire station site during the 1989 Loma Prieta earthquake is presented. In this case, values of $V_s$ were measured by cross-hole testing. Values of $V_s$ and CSR are shown in Figs. 12(a and d), respectively. These values were calculated assuming soil densities of 1.76 Mg/m$^3$ above the water table and 1.92 Mg/m$^3$ below the water table. Also assumed in the evaluation were the average values of $r_s$ originally proposed by Seed and Idriss (1971). Based on peak values of 0.16g and 0.11g recorded in two horizontal directions at the fire station during the 1989 earthquake (Brady and Shakal 1994), a geometric mean value of 0.13g was assumed for $a_{max}$. Profiles of soil type and fines content shown in Figs. 12(b and c) were based on information provided by de Alba et al. (1994) and de Alba and Faris (1996). Values of CRR were calculated assuming an MSF value of 1.19, the lower bound value recommended by the 1996 NCEER workshop (Yoid et al. 1997). The value of $K_s$ was assumed equal to 1.5 because the soil to be evaluated at this site was uncremented and <10,000 years old.

Values of $F_S$ shown in Fig. 12(e) are <1 for the depths of 4–9 m. Between the depths of 4 and 7 m, the sand contains nonplastic fines and is considered liquefiable. Between the depths of 7 and 9 m, the soil exhibits plastic characteristics and may not be liquefiable by the so-called Chinese criteria. According to the Chinese criteria, nonliquefiable clays and silts have clay contents (particles <5 μm) ≥15%, liquid limits ≥35%, or moisture contents ≥90% of the liquid limit (Seed and Idriss 1982). Thus, by the simplified $V_s$ procedure, the layer predicted likely to liquefy, or the critical layer, lies between the depths of 4 and 7 m.

Although no sand boils or ground cracks occurred at the site during the 1989 earthquake (Bennett 1994), there was a sudden drop in the fire station strong ground motion recordings at about 15 s and small motion afterward (Idriss 1990). This behavior was unlike behavior observed in recordings at other seismograph stations located on soft-soil sites in the San Francisco Bay area. de Alba et al. (1994) attributed this behavior to liquefaction of an underlying sand. A similar sudden drop in the strong ground motion recordings occurred at the Port Island Downhole Array site in Kobe, Japan, during the 1995 Hyogo-ken Nanbu earthquake (Aguiro and Ikida 1997), where liquefaction and sand boils did occur. It is probable that the 4-m-thick layer capping the level-ground fire station site, predicted not to liquefy in Figs. 12(d and e), prevented the formation of sand boils at the ground surface (Ishihara 1985). Therefore, this case history confirms the $V_s$ prediction method.

CONCLUSIONS

In this paper, a procedure is presented for evaluating liquefaction resistance through $V_s$ measurements. The procedure can be summarized in the following 10 steps:

1. From available subsurface data, develop detailed profiles of $V_s$, soil type, fines content and, if possible, soil density and penetration resistance.
2. Identify the depth of the ground-water table, noting any seasonal fluctuations and artesian pressures.
3. Calculate the values of $\sigma_s$ and $\sigma_i$ for each measurement depth at which seismic testing has been performed.
4. Correct the $V_s$ measurements to the reference overburden stress of 100 kPa using (11). The correction factor $C_v$ is limited to a maximum value of 1.4 at shallow depths.
5. Determine the value of $V_s$ for each measurement depth using (11), which is recommended for sandy as well as gravelly soils. If the fines content is unknown, assume 215 m/s.
6. Determine the value of $K_s$. The value of $K_s$ can be assumed equal to 1 if the soil to be evaluated is uncremented and <10,000 years old. If the soil conditions are unknown and penetration data are not available, assume 0.6 for $K_s$.
7. Determine the design earthquake magnitude and expected value of $a_{max}$.
8. Calculate CSR for each measurement depth below the water table using (2). The value of $r_s$ can be estimated.
from the average curve originally proposed by Seed and Idriss (1971).

9. Plot values of $V_{r1}$ and CSR and appropriate liquefaction resistance curves defined by (8), (11), and (13), with $a = 0.022$, $b = 2.8$, and $n = -2.56$.

10. Calculate the value of $FS$ for each measurement depth using (12). Liquefaction is predicted to occur when $FS \leq 1$. Liquefaction is predicted not to occur when $FS > 1$.

The procedure should be used cautiously and with engineering judgment when applying it to sites where conditions are different from the database. The case history data, and CRR-$V_{r1}$ curves, are limited to relatively level ground sites with average depths <10 m, uncemented soils of Holocene age, ground-water table depths between 0.5 and 6 m, and $V_{r1}$ measurements performed below the water table.

Three concerns when using $V_{r1}$ as an indicator of liquefaction resistance are (1) its higher sensitivity (when compared with the penetration-based methods) to weak interparticle bonding; (2) the lack of a physical sample for identifying nonliquefiable clayey soils; and (3) not detecting thin liquefiable strata because the test interval is too large. The preferred practice when using $V_{r1}$ measurements to evaluate liquefaction resistance is to drill sufficient boreholes and conduct sufficient in situ tests to detect liquefiable weakly cemented soils, identify nonliquefiable clay-rich soils, and delineate thin liquefiable strata.

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APPENDIX I. REFERENCES


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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = parameter that depends on soil structure;

\( a, b \) = parameter related to slope of CRR-Vs curve;

\( \alpha_{\text{max}} \) = peak horizontal ground surface acceleration;

\( B, B_1 \) = parameters relating \( V_{50} \) and \( (N)_{60} \);

CRR = average cyclic resistance ratio;

CSR = cyclic stress ratio;

\( C_v \) = overburden stress correction factor;

\( \varepsilon_{\text{min}} \) = minimum void ratio;

FC = fines content (particles <75 \( \mu \)m);

FS = factor of safety;

\( f(\gamma_m) \) = function of average peak cyclic shear strain;

\( G_{\text{max}} \) = small-strain shear modulus;

\( \gamma_{\text{sec}} \) = secant shear modulus at \( \gamma_m \);

\( g \) = acceleration of gravity;

\( K_c \) = cementation and aging correction factor;

\( K_{\beta} \) = coefficient of effective lateral earth pressure at rest;

\( MSF \) = magnitude scaling factor;

\( M_c \) = earthquake moment magnitude;

\( m \) = stress exponent;

\( (N)_{60} \) = SPT energy-corrected and overburden stress-corrected blow count;

\( n \) = magnitude scaling factor exponent;

\( P_t \) = probability of liquefaction occurrence;

\( P_r \) = reference overburden stress (=100 kPa);

\( r_s \) = multidirectional shaking correction factor;

\( r_S \) = shear stress reduction coefficient;

\( S_{\text{res}} \) = residual standard deviation;

\( V_S \) = small-strain shear wave velocity;

\( V_{50} \) = limiting upper value of \( V_{50} \) for cyclic liquefaction occurrence;

\( V_{51} \) = overburden stress-corrected \( V_S \);

\( z \) = depth;

\( \gamma_{av} \) = average peak cyclic shear strain;

\( \rho \) = mass density of soil;

\( \sigma_{\text{h}} \) = initial effective horizontal stress;

\( \sigma_{\text{v}} \) = total vertical (or overburden) stress;

\( \tau_{\text{av}} \) = average cyclic equivalent uniform shear stress generated by earthquake.