

# Interpretation of SDMT tests in a transversely isotropic medium

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ABSTRACT: This paper presents theoretical aspects of wave propagation in a transversely isotropic medium, aimed at providing a framework within which cross-hole (CH), down-hole (DH) and seismic dilatometer tests (SDMT) can be correctly interpreted. In particular, as an example, tests performed at the well documented Fucino site, with the source located at various distances from the sounding, indicate the capability of SDMT to detect the ratio  $G_{HH}/G_{VH}$ .

## 1 INTRODUCTION

The use of seismic methods for geotechnical site characterization is strongly motivated by the non invasive character of these tests, which preserve the initial structure of soil deposits and the major influence of all diagenetic phenomena (sutured contacts of grains, overgrowth of quartz grains, precipitation of calcite cements and authigenesis) contributing to a stiffer mechanical response, mainly at low strains (Jamiolkowski et al., 1985).

In addition, by noting that during depositional processes, soils usually experience one-dimensional deformation and the so-called initial anisotropy reflects this depositional history, it follows that a rather realistic model is, in this case, the cross-anisotropic body: the soil response is different if the loading direction changes from vertical to horizontal, but it is the same when changes occur in the horizontal plane (Hardin and Black, 1966). Seismic waves have been used to study soil anisotropy in the lab (Stokoe et al., 1980; Kuwano and Jardine, 2002). The velocity of propagation of seismic waves is influenced by both intrinsic and stress-induced anisotropy (Knox et al., 1982).

Starting from these remarks, this paper is aimed at presenting a consistent interpretation of SDMT tests, in order to detect the ratio of  $G_{HH}/G_{VH}$ .

Research currently in progress investigates the possible use of the SDMT for deriving "in situ" decay curves of soil stiffness with strain level ( $G$ - $\gamma$  curves or similar). Such curves could be tentatively constructed by fitting "reference typical-shape" laboratory curves through two points, both obtained from

SDMT: (1) the initial shear modulus  $G_0$  from  $V_S$ , and (2) a modulus at "operative" strains, corresponding to the DMT constrained modulus  $M_{DMT}$  – provided the strain range corresponding to  $M_{DMT}$  is defined. Preliminary indications suggest that the shear strain range corresponding to  $M_{DMT}$  is  $\approx 0.05$ - $0.1\%$  to  $1\%$ .

Further developments are associated to the possibility of estimating soil porosity from combined measurements of compressional and shear wave velocities (Foti et al., 2002; Foti and Lancellotta, 2004).

## 2 A REMAINDER ON WAVE PROPAGATION

A wave can be seen as a perturbation propagating with a finite speed depending on the properties of the medium, and, for this reason, within the context of continuum mechanics, a wave can be considered as a singular surface for some fields.

By considering the constitutive equation

$$\sigma_{ik} = C_{iklm} \varepsilon_{lm} \quad (1)$$

where the small strain tensor  $\varepsilon_{lm}$  is defined as the symmetric part of the displacement gradient

$$\varepsilon_{lm} = \frac{1}{2} \left( \frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l} \right) \quad (2)$$

and the equation of the motion

$$\rho \ddot{u}_i = \rho b_i + \sigma_{ik,k} \quad (3)$$

( $\rho$  is the soil density and  $b_i$  is the vector field representing the body forces per unit mass), it can be

proved, by applying the jump operator and by taking into account the continuity of the fields  $\rho, u_i, u_{i,j}, b_i$ , that the following equation is obtained

$$(C_{iklm}n_k n_l - \rho c^2 \delta_{im})a_m = 0 \quad (4)$$

Equation (4) shows that the squared speed of the propagation are the eigenvalues of the acoustic tensor:

$$A_{im} = C_{iklm}n_k n_l \quad (5)$$

where  $n_i$  is the vector normal to the wavefront and  $a_m$  is the amplitude of particle motion, or polarization vector.

To analyse the geometrical character of wave propagation, let indicate, according to Love (1944), the non vanishing components of the stiffness tensor  $C_{ijhk}$  as

$$\begin{aligned} C_{1111} &= C_{2222} = A \\ C_{3333} &= C \\ C_{3311} &= C_{3322} = F \\ C_{2323} &= C_{1313} = L \\ C_{1212} &= N \\ C_{1122} &= C_{2211} = A - 2N \end{aligned} \quad (6)$$

To give the above elastic constant a physical meaning, we write the constitutive law (1) in the following form:

$$\begin{aligned} \sigma_{xx} &= A\varepsilon_{xx} + (A - 2N)\varepsilon_{yy} + F\varepsilon_{zz} \\ \sigma_{yy} &= (A - 2N)\varepsilon_{xx} + A\varepsilon_{yy} + F\varepsilon_{zz} \\ \sigma_{zz} &= F\varepsilon_{xx} + F\varepsilon_{yy} + C\varepsilon_{zz} \\ \sigma_{xy} &= 2N\varepsilon_{xy}; \quad \sigma_{yz} = 2L\varepsilon_{yz}; \quad \sigma_{xz} = 2L\varepsilon_{xz} \end{aligned} \quad (7)$$

so that it appears that  $N$  represents the shear modulus in the horizontal plane, i.e.  $G_{HH}$ , and  $L$  is the shear modulus in the vertical plane, i.e.  $G_{VH}$ .

Let now consider the case where the normal to the plane wavefront (direction of propagation) belongs to the vertical plane ( $x_1, x_3$ ), i.e.  $n_2 = 0$ , the  $x_3$  direction assumed to be the vertical one. In addition, suppose that the particle motion (direction of polarization) is given by the vector  $\mathbf{a}(0,1,0)$ . Then equation (4) reduces to

$$(C_{2112}n_1 n_1 + C_{2332}n_3 n_3 - \rho c^2)a_2 = 0 \quad (8)$$

If  $\alpha$  is the angle between the direction of propagation and the vertical one,  $n_1 = \sin\alpha$  and  $n_3 = \cos\alpha$ , so that, by accounting for (6) and (7), the wave front propagates with a velocity equal to

$$c = \sqrt{\frac{G_{HH} \sin^2 \alpha + G_{VH} \cos^2 \alpha}{\rho}} \quad (9)$$

In particular, if the direction of propagation is coincident with the  $x_1$  axis, then the above results gives

$$c = \sqrt{\frac{G_{HH}}{\rho}} \quad (10)$$

a result which applies to cross-hole tests, when the induced shear waves is polarized in the horizontal plane (Stokoe and Woods, 1972; Ballard, 1976; Hoar and Stokoe, 1978).

On the contrary, if the direction of propagation is coincident with the  $x_3$  axis, then

$$c = \sqrt{\frac{G_{VH}}{\rho}} \quad (11)$$

Now we observe that when dealing with down-hole tests or with SMDT tests (Auld, 1977; Hepton, 1988), the measured velocity of propagation is the one given by equation (9), i.e. it depends on both shear moduli in the vertical plane and the horizontal plane. Presumed that the direction of propagation has a negligible deviation from the vertical, the velocity of propagation can be assumed to depend mainly on  $G_{VH}$ . But even in this case, the relevant aspect to be outlined is that the direction of polarization must be coincident with the  $x_2$  axis. If this is not the case, by using similar arguments, it can be proved that the velocity of propagation is a rather complicate function of 4 elastic constants, so that it is not easy to relate the measured wave velocity to soil parameters.

However, it is also apparent from equation (9) that, by performing tests at conveniently different distance between the source and the receiver, in order to change the direction of propagation, i.e. of the angle  $\alpha$ , the obtained measurements allow to obtain values of  $G_{VH}$  and  $G_{HH}$ , as it is shown in the sequel.

### 3 SEISMIC DILATOMETER TESTS AT THE FUCINO SITE

The seismic dilatometer (SDMT) is a combination of the standard flat dilatometer (DMT) equipment with a seismic module for the downhole measurement of the shear wave velocity  $V_s$ .

First introduced by Hepton (1988), the SDMT was subsequently improved at Georgia Tech, Atlanta, USA (Martin and Mayne, 1997, 1998; Mayne et al., 1999). The test is conceptually similar to the seismic cone (SCPT) (Robertson et al., 1985).

Figure 1 shows a schematic layout of the SDMT equipment used in this study.

The seismic module (Figure 1a) is a cylindrical element placed above the DMT blade, equipped with two receivers located at 0.5 m distance.

The signal is amplified and digitized at depth.

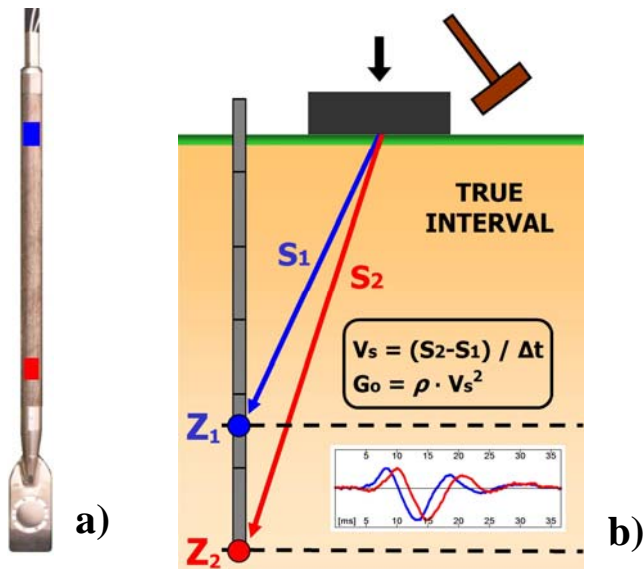


Figure 1. (a) DMT blade and seismic module. (b) Schematic layout of the seismic dilatometer test.

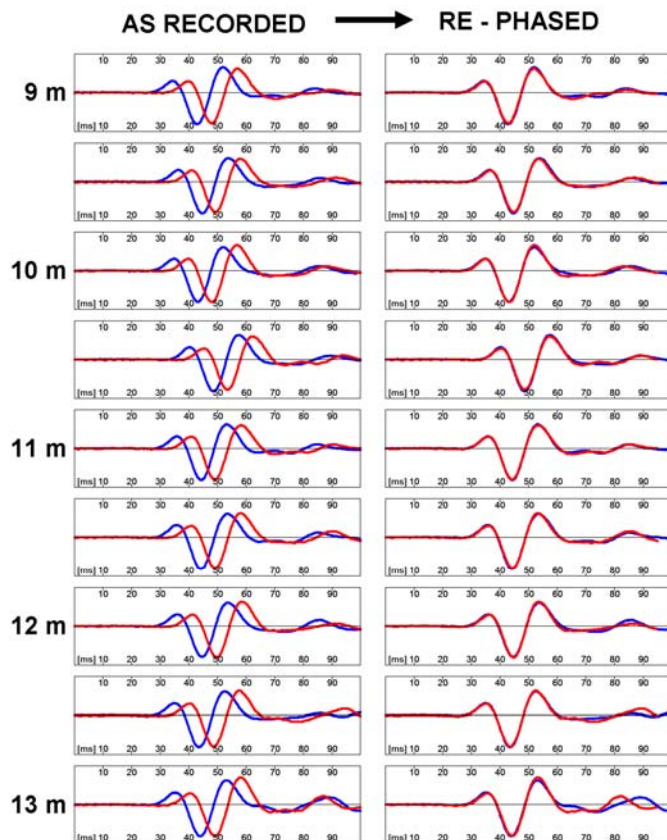


Figure 2. Example of seismograms obtained by SDMT at various test depths at the Fucino site (as recorded and re-phased according to the calculated delay)

The shear wave source at the surface is a pendulum hammer, of approximately 10 kg weight, which hits horizontally a steel rectangular base pressed vertically against the soil and oriented with its long axis ( $\approx 0.8$  m) parallel to the axis of the receivers, so that they can offer the highest sensitivity to the generated shear wave.

The "true-interval" test configuration with two receivers avoids possible inaccuracy in the determination of the "zero time" at the hammer impact, sometimes observed in the "pseudo-interval" one-receiver configuration. Moreover, the couple of seismograms recorded by the two receivers at a given test depth (Figure 2) corresponds to the same hammer blow and not to different blows in sequence, not necessarily identical. Hence the repeatability of  $V_S$  measurements is considerably improved – observed  $V_S$  repeatability about 1 m/s.

The shear wave velocity  $V_S$  (Figure 1b) is obtained as the ratio between the difference in distance between the source and the two receivers ( $S_2 - S_1$ ) and the delay of the arrival of the impulse from the first to the second receiver ( $\Delta t$ ).

$V_S$  measurements are obtained every 0.5 m of depth.

Seismic dilatometer tests were performed in 2004-2005 at the site of Fucino (Italy), a well-documented research test site, extensively investigated at the end of the '80s by means of several in situ and laboratory tests carried out by various research groups. Results of this investigation and a detailed characterization of the site can be found in AGI (1991).

The soil is constituted by a thick deposit of soft, homogeneous highly structured  $\text{CaCO}_3$  cemented lacustrine clay of high plasticity.

The clay deposit is lightly overconsolidated. Based on geological evidence, this overconsolidation is most likely due to structure/aging, in particular to secondary consolidation and post-depositional diagenetic bonds caused by  $\text{CaCO}_3$  cementation. In the upper few meters of the deposit, overconsolidation may be due in part also to groundwater level fluctuation (the water table is about 1 m below the ground surface).

The significant diagenetic bonds due to  $\text{CaCO}_3$  cementation have a strong influence on most of the soil parameters obtained from the interpretation of in situ and laboratory tests in the Fucino clay (AGI, 1991). E.g. oedometer tests suggested a quantitative link between  $\text{CaCO}_3$  content and OCR. A dependence of the undrained shear strength  $c_u$  on  $\text{CaCO}_3$  content was evidenced in particular by UU triaxial compression tests.

The values of the small strain shear modulus  $G_0$  resulting from both laboratory and in situ seismic tests also appeared to be influenced by the  $\text{CaCO}_3$  content.

Figure 3 shows the most significant profiles obtained by SDMT at the Fucino site.

The basic DMT parameters – material index  $I_D$  (soil type), constrained modulus  $M$ , undrained shear strength  $c_u$  and horizontal stress index  $K_D$  (related to stress history) – were obtained using current correlations (Marchetti, 1980).

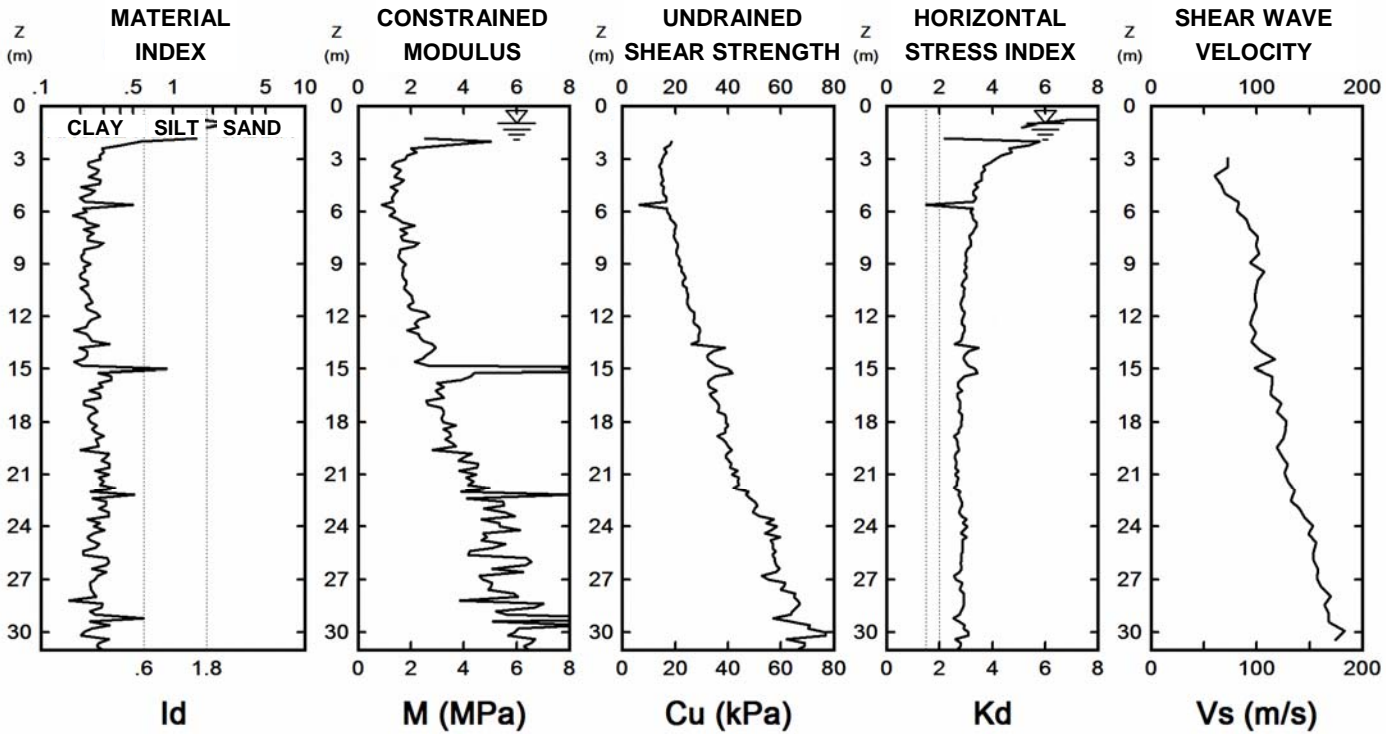


Figure 3. SDMT profiles at the Fucino site

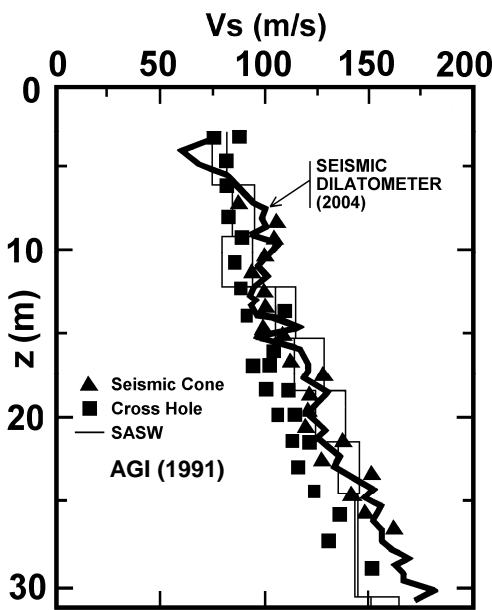


Figure 4. Comparison of  $V_S$  profiles obtained by SDMT and by other in situ seismic tests at the Fucino site (AGI, 1991)

The values of the horizontal stress index  $K_D$  (Figure 3) are  $\approx 3$  to 4, constant with depth. As indicated in TC16 (2001), if a geologically NC clay has  $K_D > 2$ , any excess of  $K_D$  above the value  $K_D \approx 2$  (lower bound value for genuinely NC clays) indicates the likely existence of cementation/structure/aging. However the NC condition can be easily recognized, despite  $K_D > 2$ , because  $K_D$  does not decrease with depth as in OC deposits.

The profile of the shear wave velocity  $V_S$  obtained by SDMT, plotted in Figure 3, is also shown in Figure 4, superimposed to profiles of  $V_S$  obtained by seismic cone penetration tests (SCPT), cross-hole and SASW in previous investigations (AGI, 1991). The comparison in Figure 4 shows that  $V_S$  obtained by SDMT is in good agreement with  $V_S$  obtained by other methods.

#### 4 ANISOTROPY RATIO FROM RESULTS AT THE FUCINO SITE

In order to explore the possibility of using Equation 9 to obtain  $G_{HH}$  and  $G_{VH}$  in anisotropic media a testing campaign has been planned at Fucino test site. To determine the two shear moduli, at least two independent evaluations of shear wave velocity are needed with different angle of incidence with respect to the receivers.

The experimental data have been collected using the usual SDMT configuration, repeating then the test for two additional shot locations as shown in Figure 5. The sources are placed along a straight line starting from the position of the SDMT probe and are orientated perpendicular to the line itself in order to detect primarily horizontally polarized shear waves (Figure 5a). The shear wave velocity obtained in each testing configuration has been associated to the angle of incidence corresponding to the hammer position and to the intermediate point in between the two receivers (Figure 5b).

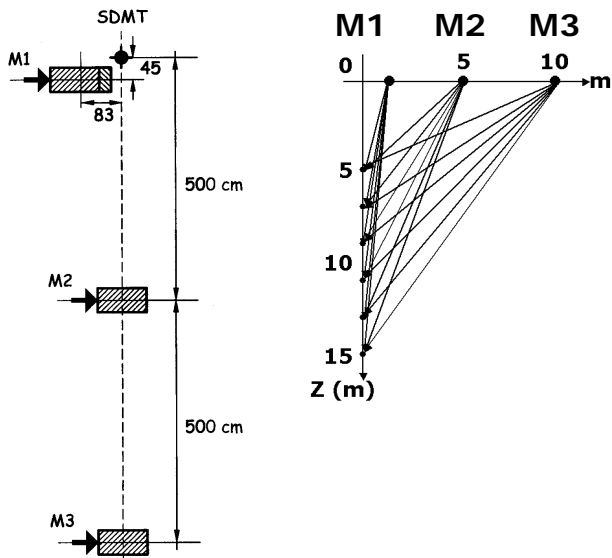


Figure 5. Test setup: a) Plan view b) Ray paths

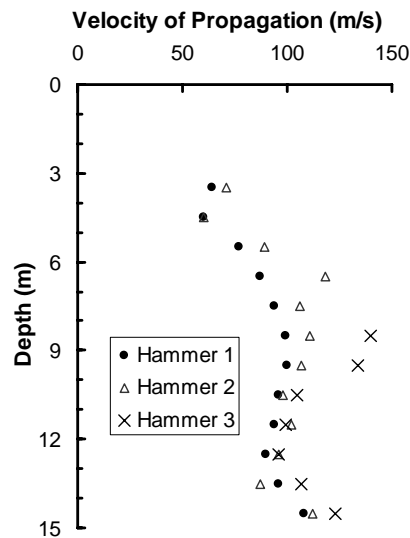


Figure 6. Shear wave velocity profiles

Shear wave velocity measurements have been performed at 1m interval from 3.5 to 14.5m, but the data for depth 3.5m to 7.5m for the third hammer (M3) are not used in the following because they showed unusual results.

The shear wave velocity profile obtained using a true interval interpretation of experimental data is reported in Figure 6. As explained in the previous section, these velocities have to be regarded as intermediate values between those pertinent to vertically traveling-horizontally polarized shear waves and horizontally traveling-horizontally polarized shear waves. Hence, assuming homogeneity of the medium in between the receiver position, in the case of an isotropic medium, the three velocities should coincide. The detected differences can be interpreted in the framework of anisotropic linear elasticity.

For depths from 3.5m to 7.5m only two measurements of  $V_S$  were available (from hammers M1 and M2), hence the shear moduli have been obtained directly by using Equation 9 and solving the system of 2 equations in two unknowns for each depth. For depths 8.5m to 14.5m, since three measurements were available for the determination of two parameters, an optimization procedure has been adopted, selecting the two values of the moduli at each depth such that the minimum difference in the least square sense was obtained between the experimental values and the velocities predicted with Equation 9 for the three available measurements.

The values of the shear moduli and their ratio are reported in Figure 7. Most of the results show a ratio of the two moduli ranging between 1 and 2, that seems reasonable for the site characteristics. Two values out of trend ranging between 3 and 4 are obtained for depth of 6.5m and 8.5m. There seem to be no coherent explanation for these values, which have been considered as experimental scatter.

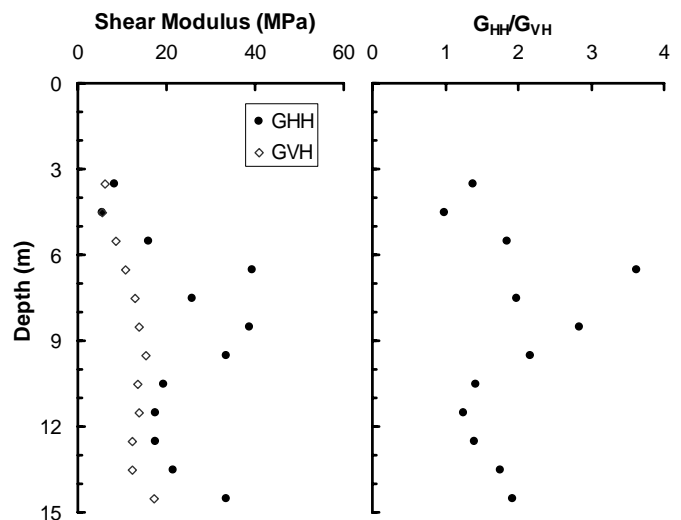


Figure 7. Shear Moduli obtained from measured  $V_S$

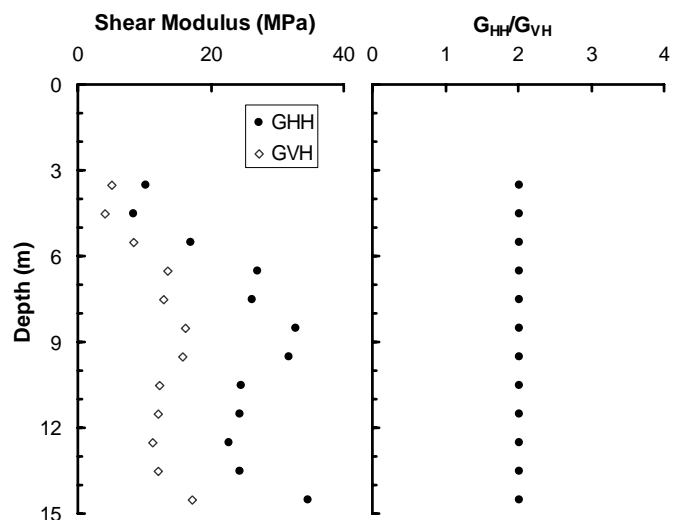


Figure 8. Shear Moduli obtained from measured  $V_S$  with the constraint  $G_{HH}/G_{VH} = \text{constant}$

Considering the peculiarity of the experimental site, consisting of a very homogeneous soft clay, a second interpretation was attempted, imposing the condition of constant ratio between the shear moduli ( $G_{HH}/G_{VH} = \text{constant}$ ). The ratio was one of the parameters in the optimization procedure together with one of the two moduli at each depth. The results are reported in Figure 8 and show a global value of  $G_{HH}/G_{VH}$  equal to 2.0.

## 5 CONCLUSIONS

From the operative viewpoint the described investigation has evidenced the following features of the seismic dilatometer:

- Simplicity of operation.
- High quality of the signals.
- Accurate determination of the shear wave velocity  $V_s$ .
- High repeatability.

From the interpretation viewpoint the investigation has shown that, if the SDMT is performed by placing the source, for each probe depth, both adjacent to the sounding and at conveniently different distances, the obtained SDMT measurements allow, on the basis of wave propagation theory for anisotropic media, to evaluate anisotropy, in particular to obtain values of  $G_{VH}$  and  $G_{HH}$ .

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